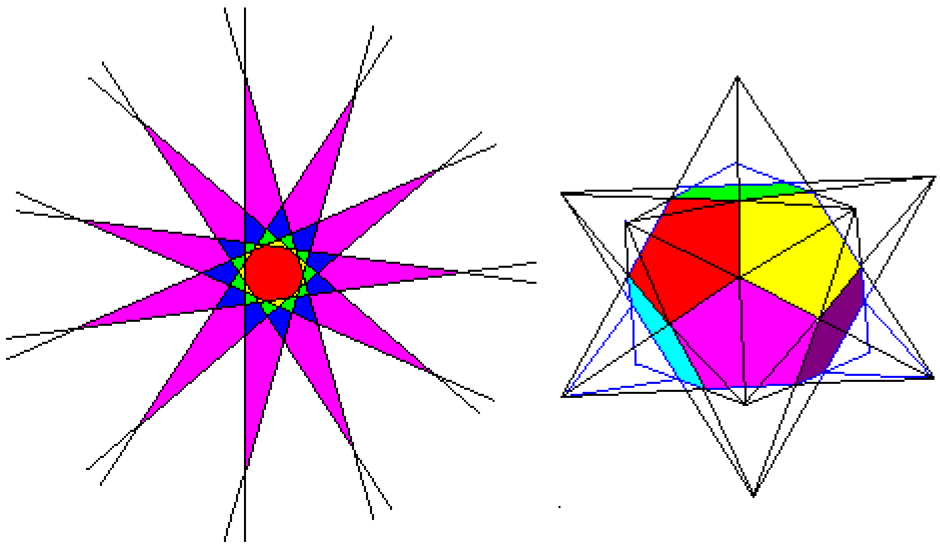
CHAPTER 10 - STELLATIONS OF POLYHEDRA



Adapted from: http://www.uwgb.edu/DutchS/symmetry/stellate.htm

If you extend the edges of a polygon, they intersect to form a star. In the left diagram above, an 11-sided polygon has its edges extended to form star polygons. Each possible star is denoted by a different color. Beyond the outermost star, all the lines diverge, and never intersect again. Likewise, we can extend the faces and edges of a polyhedron. If we do this to a dodecahedron, as shown at right, the faces become stars and we obtain the star polyhedron shown. This process is called *stellation.* (Steven Dutch, Natural and Applied Sciences, University of Wisconsin - Green Bay)

Objective:
To create several stellated polyhedra.

Useful Vocabulary:
Isosceles triangle
Johannes Kepler
Symmetric
Stellation
Stella

Materials:
Standard white paper
Several large sheets of 22 x 29-inch white poster-board (cut in half to the more manageable dimensions of 22 x 14 1/2-inches),
A ruler
Sharp No. 2 pencils

A quality compass
A pair of scissors
Cellophane tape
An eraser
1/2-inch x 3 1/2-inch white label strips (recommended.)

The Stella Octangula

This is by far the easiest of all stellations to make. The name means “eight-pointed star,” given to it by its discoverer, the great scientist, mathematician and astronomer, Johannes Kepler early in the 17th century. It is nothing more than a stellated octahedron which, having eight faces, will also have the same number of star points.

To construct a Stella Octangula, you may follow either of two quick processes.

1. Follow the instructions given in **Chapter 5** to make an octahedron, **Figure 10.1** and then make eight tetrahedra, **Figure 10.2** using the same sized equilateral triangles. Affix one tetrahedron to each face of the octahedron by aligning a triangular face of each, and glue in place. See **Figure 10.3** for the completed Stella Octangula.

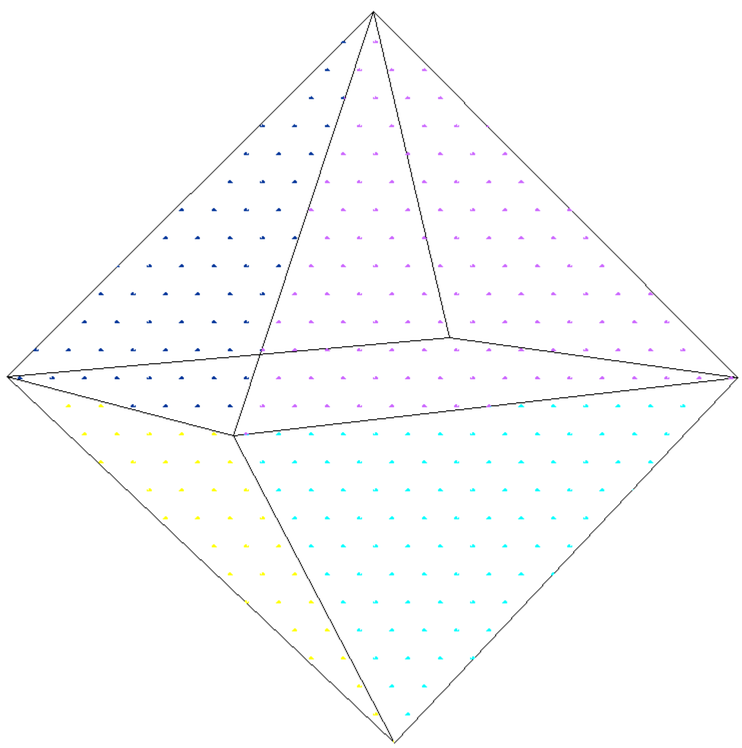


Figure 10.1 octahedron from Chapter 3

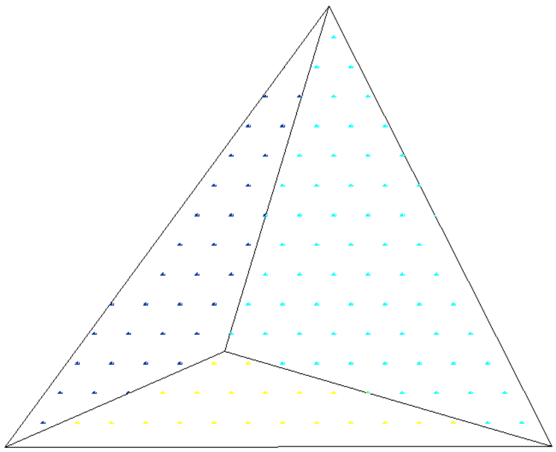


Figure 10.2 tetrahedron from Chapter 3

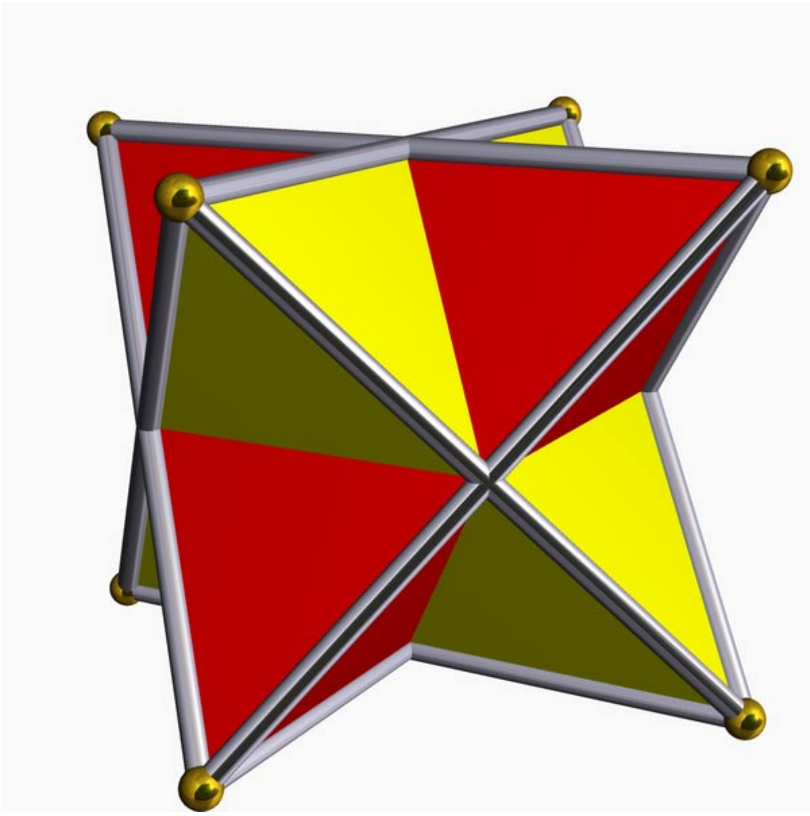


Figure 10.3
(http://en.wikipedia.org/wiki/Image:Compound_of_two_tetrahedra.png)

2. When finished, study the shape, and if you look carefully, you will see that the Stella Octangula is actually two large intersecting tetrahedra! Furthermore, depending on how you hold it and look at it (especially with one eye closed for a "two dimensional" effect), you will detect the shape of the six-pointed "Star of David."

The Small 12-pointed Stellated Dodecahedron

This construction is often referred to as the "Radiant Star," has twelve projecting points, it is indeed a star form. Such star forms are called "stellations," from the Latin word for star: "Stella," and "stellated" meaning a star form based on a solid by extending its surface planes and lines into space. In this case, our stellation is based on the solid known as a dodecahedron, **Figure 10.4**, one of the five Platonic solids, which we introduced in Chapter 5.

Dodecahedron
Figure 10.4 from Chapter 5

With its 12 pentagonal sides, the dodecahedron serves as the "core" of this star form. Affixed to each side is a star point consisting of five Isosceles triangles. If you had to cut out all the individual pieces, you would need 12 pentagons and 60 Isosceles triangles (an Isosceles triangle has two equal sides). The length of the base of each isosceles triangle must be equal to each side of the regular pentagons used. Fortunately, we will be taking some major short-cuts! Once the various pieces have been made, its assembly is relatively easy, and the finished product quite attractive.

1. The Isosceles triangle and the pentagon required for this exercise are directly related to each other. Follow the instructions given in **Chapter 5** to construct a pentagon with the dimensions desired.

2. Once you have created the pentagon, use your straight-edge to extend all of the five side lines into space in both directions, and you will soon see that these lines come together at specific points outside the pentagon, forming a “pentagram” (“penta” in Greek means “five”), a five-pointed star, the very type used on the American flag. See **Figure 10.5**. Each one of these isosceles triangles formed is of the correct size for our construction.

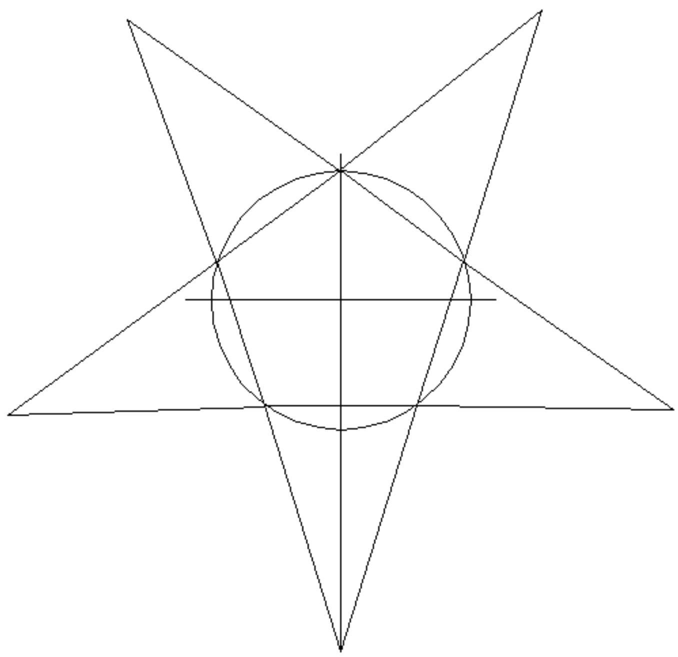


Figure 10.5

3. Carefully cut out the pentagon and one of the triangles. These will be used as templates in designing the dodecahedron and all of the star-points.

4. Using the pentagon as a guide, design and assemble a dodecahedron according to the pattern given in **<u>Chapter 5</u>**. In this case, it is better to use glue or paper label strips to bind the edges, since the star points will not adhere to a taped edge.

5. Use the isosceles triangle template to fashion the pattern given in **Figure 10.6**, adding the necessary glue tabs as indicated. You will need twelve of these, each containing an arrangement of five triangles laid point to point.

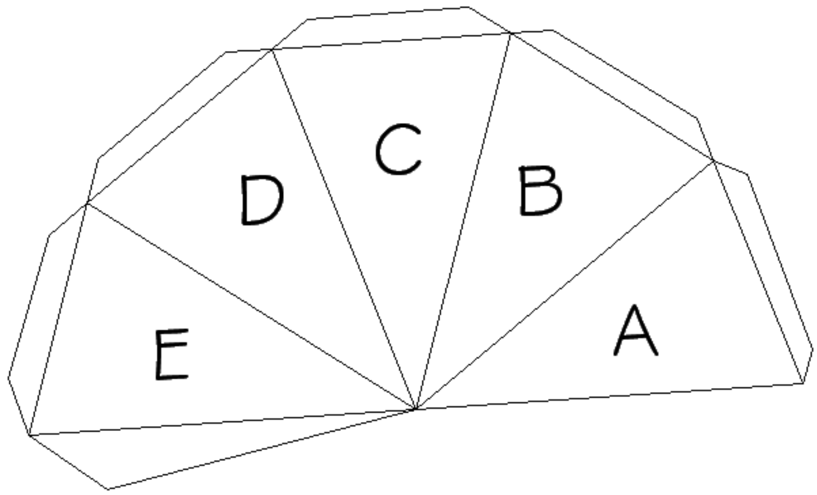


Figure 10.6

6. The first piece completed can serve as a template for all the others, but be sure to transfer all the lines needed for scoring and folding. The notches between the glue tabs and the tip can serve as reference points when you use a straight-edge and compass point. Cut and fold so that the glue tab attached to triangle E fall behind triangle A and glue or tape along the edge.

7. The five glue tabs at the base should fold inward. When assembled, you will have one of the twelve star points of a Small 12-pointed Stellated Dodecahedron. See **Figure 10.7**. Construct all twelve.

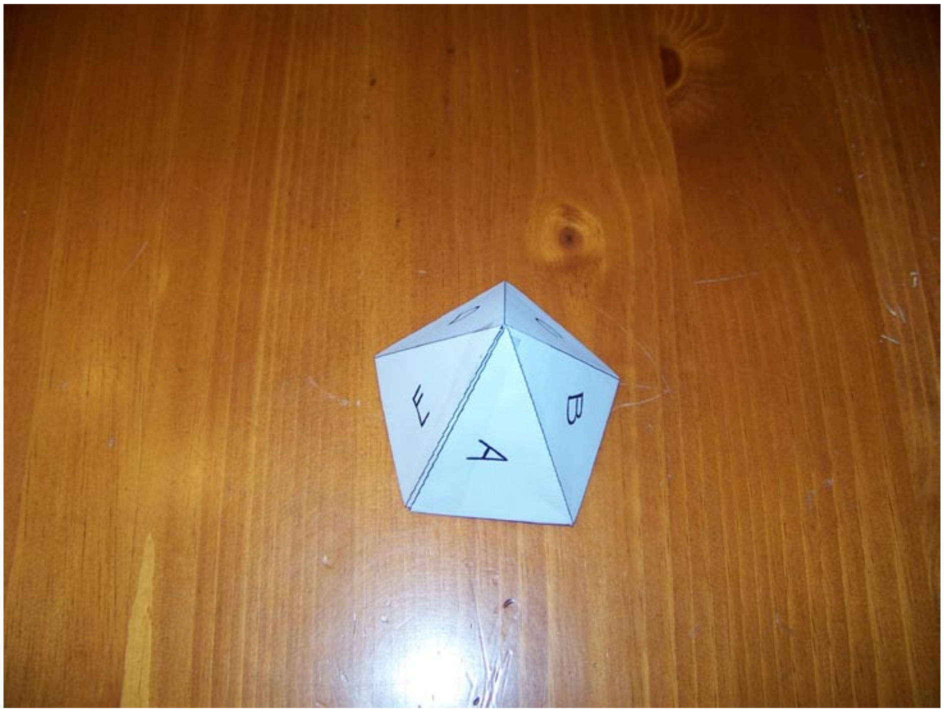


Figure 10.7

8. By placing a star point on each face of the dodecahedron, you will see how they are designed to fit. To assure proper alignment, glue one tab of a star point along the edge of a pentagon face. A small piece of clear tape will help to hold it in place until the glue has dried. Then glue the next tab in place. When this tab has dried, the remaining three tabs can be glued all at once, held in place by a few more short pieces of tape. When the glue has dried, carefully peel off the tape. Repeat this process until all 12 points are in place. **Figure 10.8** shows the finished product.

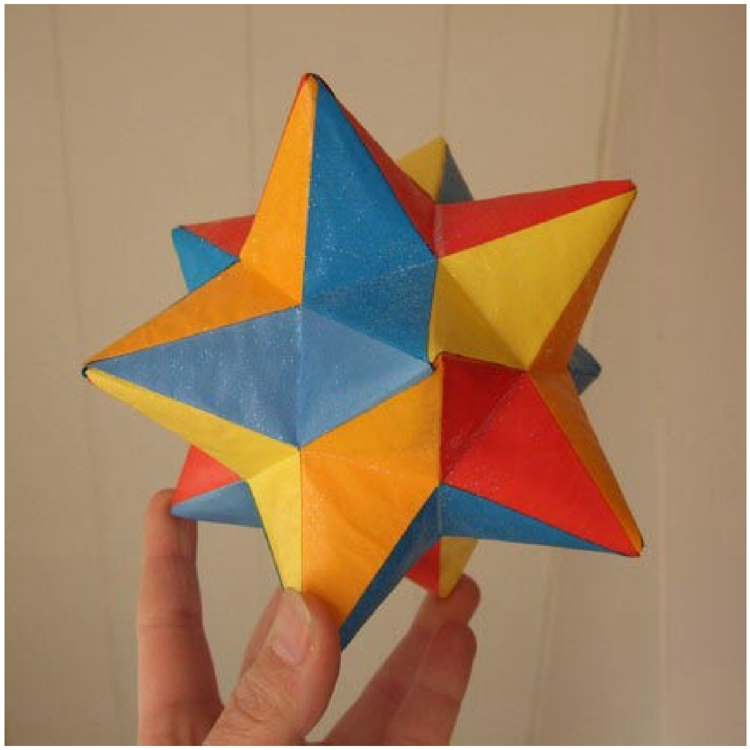


Figure 10.8

Additional exercise:

If you wish to display your finished star by hanging it up, install a piece of string through one of the star points during construction, held inside by a paper-clip or a toothpick.

A much larger version of this same star can be constructed as a class project. Pentagons with 12” edges and similarly proportioned isosceles triangles will result in a spectacularly grand “radiant star” about 4-1/2 feet in diameter. For this purpose, the 12 pentagons and 60 triangles should be cut out individually by adults using a heavy cardboard stock. Accuracy is critical, especially with these big pieces! Remember, the Canton Museum of Art, Canton, Ohio, may be able to help with this process.

Assembly in the large format is pretty much the same as the small. Now, however, we must use two-inch wide clear tape, and all work should be done on the inner surface of the structure as much as possible. Assemble the dodecahedron described in Chapter 5 first, with all edges fitting snugly together.

The large triangles should be arranged on a flat surface in groups of five (face down) to match the pattern in **Figure 10.6**. Tape triangles snugly together with strips of wide tape running the full length of the seam. These tape strips will provide the “hinge” or fold when the five triangles are looped to form the five-sided point.

To complete each five-sided point, lift triangle A and loop it over so that it falls into alignment with the edge of triangle E. Once the point is complete and sits upright by itself, reinforce all the seams on the outside by running long tape strips along the full length.

Repeat this process with all the remaining triangles until you have constructed twelve five-sided points. For the exterior taping, you might consider using a white tape, which will give your finished star a very professional look.

Set the first star point on the top face of the dodecahedron, carefully aligning one triangle edge with the edge of the pentagon face. Tape the edge flush to the dodecahedron, and repeat the process on the four remaining sides of the star point, one at a time. Be sure to rub the tape down with your fingernail or some other hard object to get a tight bond. With only one point affixed, the construction will look something like a giant geometric ice cream cone.

Affix the next point adjacent to the first, which will create a tight valley between the two. Align and tape all of the other four sides of this point first, one at a time, saving the valley for last. To complete this section, take a piece of tape the length of the valley stretched between both hands, sticky side down. Carefully fold it along its full length, sticky side out. Thus folded, place the tape very tightly into the valley area going in as far as possible so that it covers the seam, but has not yet adhered firmly to either surface. Press your finger along one side of the tape, causing it to stick firmly to one side, and gently forcing the tape into the seam as tightly as possible. Then, with the tape fully bonded along one side, run your finger along the other side. You'll be surprised what a snug fit you can get in this way.

Next, add a third point in the space between these two, which will give you three easy joints to tape, and two valleys. When #3 point is in place, note its resemblance to a giant tooth!

Proceed to install the remaining points, setting each new one into the groove between the two previously installed ones. You will eventually find that the last few star points will have more valley joints, and the last point will require five valley joints. But if you follow the "folded tape" system detailed above, every star point should remain in place as long as the finished star isn't handled carelessly.

Be sure to stop occasionally to study your work as you go along. For example, note how each finished face of the construction looks like an "American Star." **Figure 10.5** shows what the finished star will look like, only this one will be much bigger.

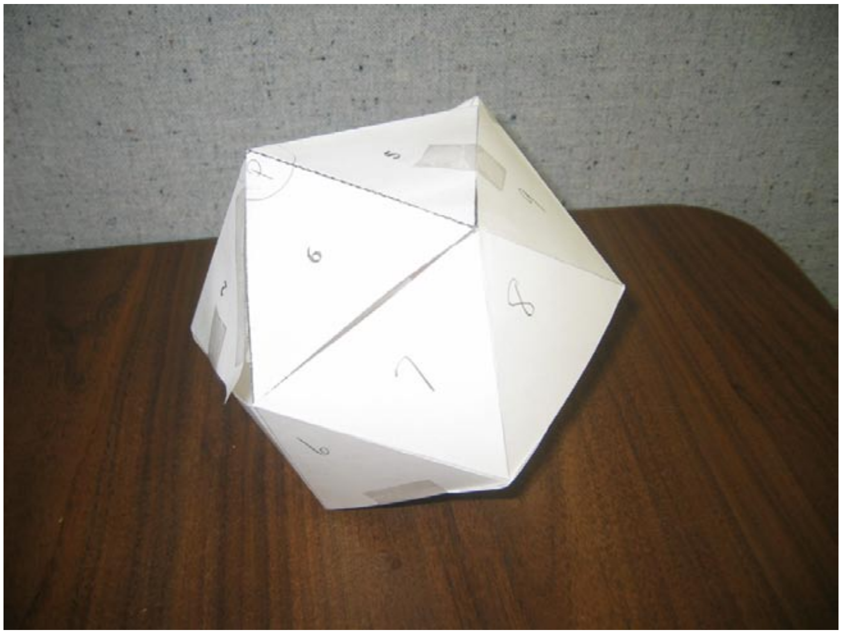
For even greater permanence, you may wish to bond all the edges of the dodecahedron and star points with some kind of glue. While this is perhaps best of all, resulting in a very sturdy long-lasting star, this process will take much more time and patience. If you decide to do this, it is recommended that heavy paper strips be glued in place to cover all the seams inside and out, which will greatly strengthen the star.

Your giant stellation makes an excellent interior holiday decoration for the hallway, lobby, or Gym, since you can draw or paint on its many exposed surfaces. If you want to suspend it, remember to install a strong piece of heavy twine or fishing line through one of the points going all the way into one face of the interior dodecahedron. Here, it should be held in place by a small piece of wood, about 1 x 2 x 8-inches. One word of caution – these stars are NOT fire-proof, so don't expose them to heat, electricity, or other potential fire hazards.

One other interesting note – the star points can actually be exaggerated for dramatic or decorative effect by extending the length of the isosceles triangles which make up all the points. The resulting star will no longer be a legitimate geometric shape, but will produce a magnificent star! The base of each triangle must match the edge of the pentagon face of the dodecahedron, while the isosceles sides can be extended perhaps 25%. But be advised – even the slightest lengthening will make a big difference.

The Great 20-pointed Stellated Dodecahedron

For this construction, you must go back to the design given in Chapter 3 for the icosahedron, the Platonic Solid made up of 20 equilateral triangles. See **Figure 10.9**. This is a tougher model to make because of its complexity, but these instructions should see you through. But wait--if this is to be the Great Stellated Dodecahedron, why are we building upon an icosahedron? There is a unique relationship between these two Platonic Solids--as there is between the square and the octahedron, Chapter 3. If you connect the midpoints of all the faces of a dodecahedron by straight lines on the interior, the resulting shape would be an icosahedron--and vice versa.



Icosahedron
Figure 10.9 from Chapter 3

Here's another way of understanding the concept: you could build a dodecahedron around an icosahedron, and if the fit was "just right," the vertices of the icosahedron would touch each interior face of the dodecahedron at the mid-point. The same is true the other way around. (That's also true of the square and the octahedron. The only shape that can occur within a tetrahedron is . . . another, smaller tetrahedron. **Chapter 3**)

1. The very same type of isosceles triangle is needed here as in the Small version, so you may use the same template to design the star points. In this model, however, only three are needed because each face of the icosahedron is a triangle. See **Figure 10.10**.

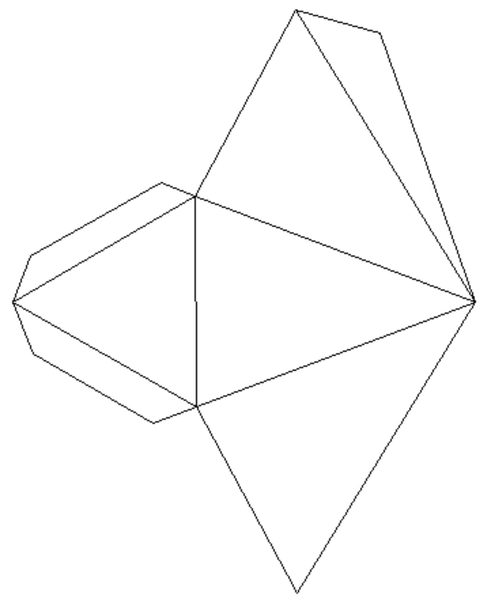


Figure 10.10

2. Unlike the template for the previous stellation, this one has a base, which will be a big help in assembling a much more complex model. That's the good news. The bad news is, that you need twenty of these star points. Follow the net, using the glue tabs provided. Make one three-triangle template and use this to lay out the other nineteen. Score all interior lines, cut out, fold and glue.

3. Construct an icosahedron net using instructions in **Chapter 3**. The measurement of the base of your isosceles triangle will be the length of each side of your equilateral triangle. Having scored and cut out the net, use glue or paper strips on the exterior to bond the edges.

4. When ready, take any one star point (base glued in place), carefully align it with any one face of the icosahedron, and glue it to the surface. When dry, add the second next to it, then the third, and on to the end.

If this process gives you any trouble, or if you are not fully satisfied with the results, here are two other possibilities:

1. Snip off the base of every point and glue each triangle point directly to an icosahedron face along the edges.

Or

2. Do not use the icosahedron as an interior support. Instead, read on: this finished star form subdivides into three specific parts. At the top, the group of five star points forms a “crown” with a pentagonal base. Around the center is a band of ten points, and there is a bottom or “foot” of five star points just like the top. Design two pentagons with edges equal to the base of a five-point assembly. Then, assemble a series of 10 star points in a circle by taping them together with a flap over-lapping two triangle edges as in **Figure 10.11**. A pentagon installed top and bottom will stabilize this central band of ten points.

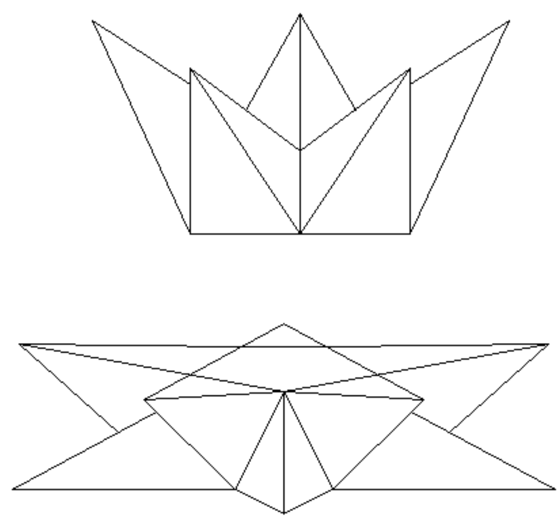


Figure 10.11

Join five points in a circle in the same way, as shown in **Figure 10.11**, and glue them in place along the outer edge of the upper pentagon. Do the same with the remaining five pieces for the bottom, and your star is finished. See **Figure 10.12**. When you look at your finished Great Stellated Dodecahedron, you will see that the points are in groups of five arranged in a pentagon, implying the “ghost” of a dodecahedron inside.

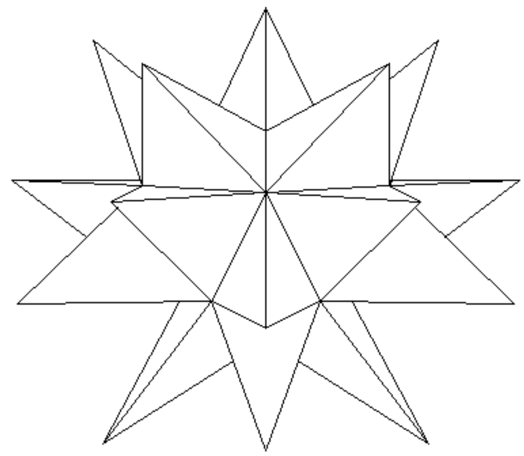


Figure 10.12

Third Stellation of the Icosahedron

This is a fast and simple construction, and furthermore, it is fun to assemble, and the result is quite surprising. When finished, this stellation will immediately call to mind the dodecahedron, except that each polygon face will consist of five equilateral triangles.

1. Using the hexagon net you designed in **Chapter 3**, (see **Figures 3.6 and 3.7)**, make 12 pentagonal pyramids (five equilateral triangles), and tape any two together along one common edge.

2. Installing a third will require that you decide on the shape your form will eventually take—should the pyramids bulge outward, or should they "dimple" inward? The answer is, if you are making the legitimate geometric form, they must "dimple" inward. This seems like a move in the wrong direction, but in the end, the Third Stellation is visually quite pleasing to the eye. See **Figure 10.13**.

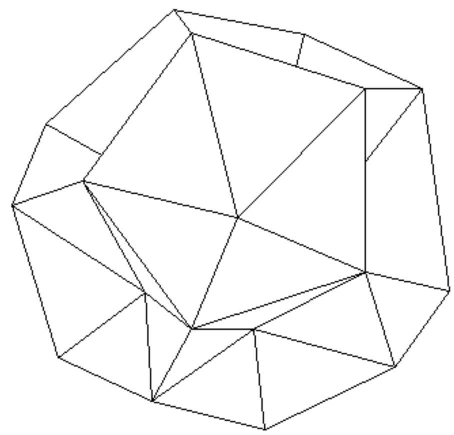


Figure 10.13

You may wish to repeat the process and produce the opposite form—not a legitimate stellation, but still quite beautiful, resembling the head of a scepter. Because this polyhedron is comprised entirely of equilateral triangles, it qualifies as a deltahedron, as does its opposite form, and can be grouped with those discussed in **Chapter 3**.

Many other stellations are possible, of increasing complexity and great beauty, and merit your attention as your skills increase. If the star-making bug has bitten you, I encourage you to purchase Polyhedron Models by Magnus J. Wenninger (Cambridge University Press), which contains instructions for dozens of increasingly complicated stellations, some requiring hundreds of pieces! There are months—and years—of hard work (and pleasure) ahead for you!