CHAPTER 11 - COLLAPSOIDS: SHAPES THAT WON'T HOLD THEIR SHAPE

Just about the time when everyone starts to think that there is nothing more to be discovered about polyhedra, some scholar comes across something new and unexpected. Such is the case with a group of unstable forms now generally referred to as "collapsoids." The first collapsoids seem to have been discovered by Professor Jean J. Pedersen of the University of Santa Clara, in California and we are indebted to her for several ideas presented in this section.

Most simply, we might define a collapsoid as any unstable geometric construction which retains it shape as long as certain sections are held together by hand, tape, or paper clips. When released, these shapes will "collapse" and completely lose their form, and in some ideal cases will flatten completely. Such a construction can be reconstituted quickly by picking it up by certain sections, bringing these together, and holding them in place. One is reminded of party or holiday decorations made of paper which come flat, but open up to become snowmen, a bells, turkeys, or other dimensional shapes--except that collapsoids are much more complicated.

Objective:
To create three-dimensional "collapsoids".

Useful Vocabulary:
Collapsoid
Dipyramid
Dodecahedron
Equatorial Collapsoid
Icosahedron
Octahedron
Polar
Polar Collapsoid
Skew
Pseudo-Rhombic Dodecahedron
Pseudo-Rhombic Icosahedron
Pseudo-Triacontahedron

Materials:
Standard white paper
Several large sheets of 22 x 29-inch white poster-board (cut in half to the more manageable dimensions of 22 x 14 1/2-inches),
A ruler
Sharp No. 2 pencils
A quality compass
A pair of scissors
Cellophane tape
An eraser
1/2-inch x 3 1/2-inch white label strips (recommended)

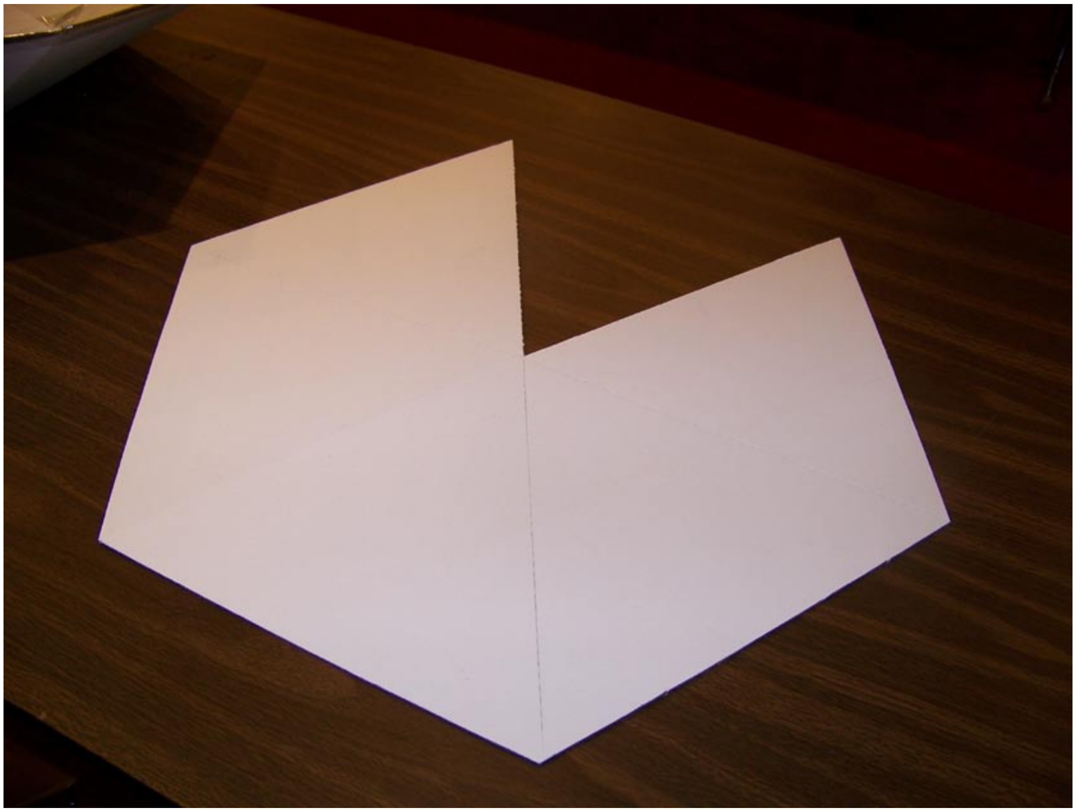
Steps:

First, we will look at a relatively simple collapsoid design based on the equilateral triangle.

1. On a sheet of poster board, design a hexagon based on a circle, (see **Chapter 3**) and subdivide it into 6 equilateral triangles scoring the lines drawn with a compass. Make three more of these to the same size, score all the lines between the triangles, and cut out the hexagons.

2. Cut out one entire triangle in each hexagon. Gently fold each scored line to give it some flexibility, then, return to its original form. (If using individually-cut triangles, tape them together--one side only--in a hexagonal arrangement of five, making four sets.)

3. Sandwich two hexagons together (taped edges inside) so that the cut-out area is aligned on both pieces, and apply tape along all of the five common outer edges. You should be able to manipulate this finished piece like a “fish mouth,” which will open and close when pressure is applied around the edge. In fact, when the “mouth” is closed, the construction will have the appearance of a pentagonal dipyramid; when open and flat, it will look like a hexagon minus one triangle (**Figure 11.1**)



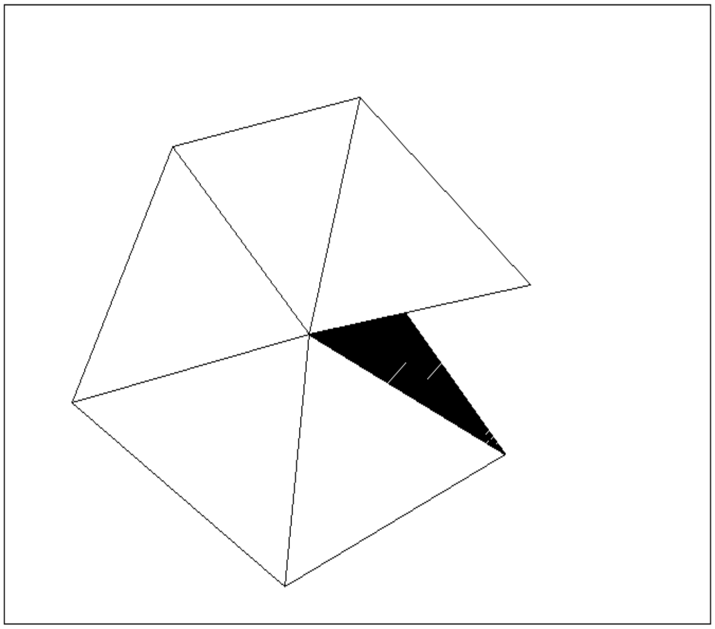


Figure 11.1

4. A bit of "conceptualizing" will help at this point. Imagine another piece just like the one you made, and connecting the two "mouths" at a 90-degree angle. It would seem that the next logical step would be to actually make the second unit, using the two remaining hexagonal pieces.

5. Affix the 4 remaining edges of the "mouths." The finished construction will look much like two flying saucers which have crashed into each other. Compress one saucer, and the other will bulge out, and vice versa--thus the name "collapsoid" (**Figure 11.2).**

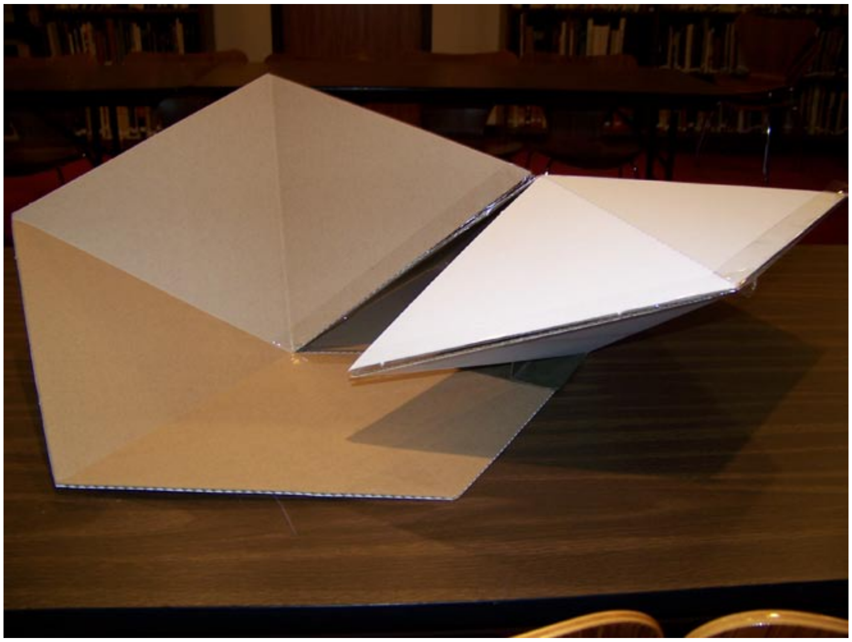


Figure 11.2

Professor Pederson's 12-celled Collapsoid, the Pseudo-Rhombic Dodecahedron

Prof. Pedersen's 12-celled collapsoid is based on the cube and the octahedron, since both have 12 edges. In effect, this collapsoid model substitutes one baseless four-sided pyramid (flattened) for each edge of the cube or the octahedron. In the case of the cube, the resulting construction is called an equatorial collapsoid, and for the octahedron, it is a polar collapsoid. If all of this sounds too confusing, you may proceed to make the collapsoid anyway, and appreciate its properties just for the fun of it. By the way, Prof. Pedersen calls this a "pseudo-rhombic dodecahedron."

1. You can start by designing a hexagon in **Chapter 3** or just put together a series of four equilateral triangles around a point. Make 12 pyramid forms out of these series of 4 equilateral triangles which will be open at the bottom. The template for such a cell is given in **Figure 11.3,** with a glue tab, along with an illustration of a finished cell.

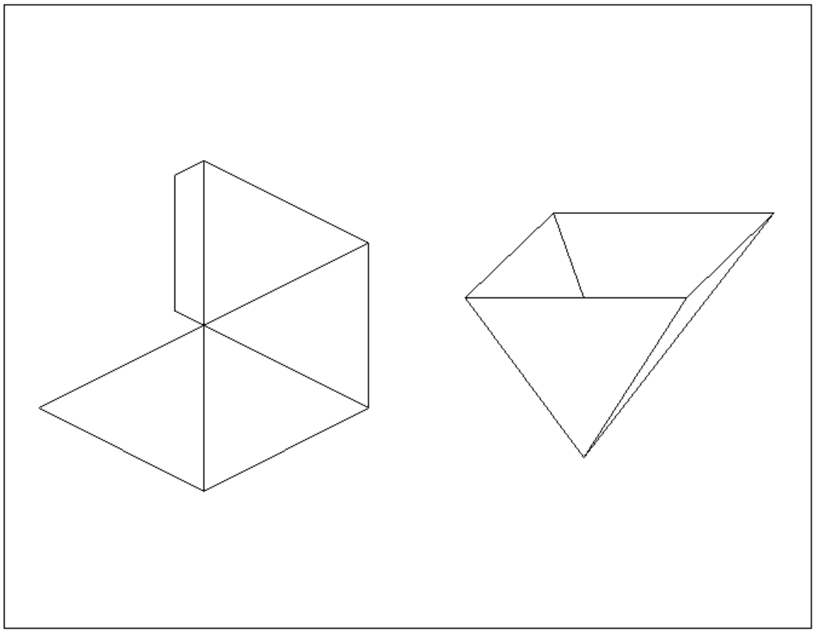


Figure 11.3

2. Tape six of these cells in a row, open base up, with a strip of tape laid over the common edges. When this step is completed, your construction will look like six up-side-down pyramids in a row. See **Figure 11.4**.

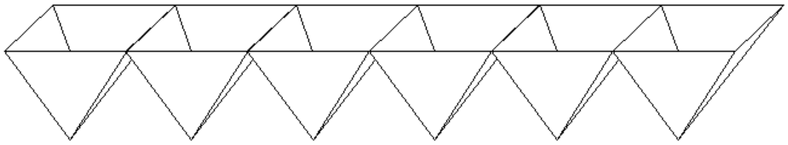


Figure 11.4

3. Skew cells 1 and 2 in the manner displayed in **Figure 11.5** to allow cell 7 to be taped in place, one edge to the first cell, and another to the second. In the same way, attach cell 8 along the sides of cells 3 and 4, and cell 9 with 5 and 6. Three cells remain, 10, 11, and 12, each of which can be installed on the other side of the set of six in the same fashion. Cell 12, the last one, will attach to cell 6 along one edge only.

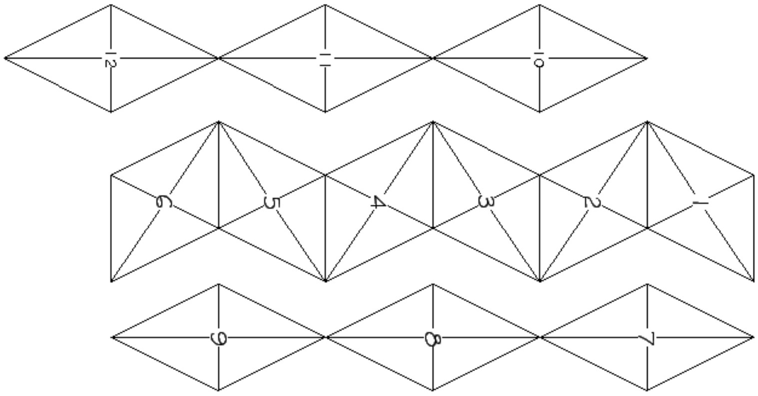


Figure 11.5

4. Here, I am indebted to Prof. Pedersen for her net, and for one of her ideas on how to connect the pieces. If this assemblage is folded around upon itself as in **Figure 11.6** so that AB joins to A’B’ and BC to B’C’, we have an equatorial collapsoid (folded along the length or “equator” of the piece). These joints must not be permanent, but must be held together artificially in some way to keep the “collapsoid” from, well, collapsing. She

suggests holding it together by a complex system of folded paper hinges through which paper clips are passed, a precaution against forcing the triangular faces into direct contact with each other. I suggest using a little tab of cellophane tape, which can just as easily be released or cut, and replaced as needed. When these tabs are released, the form collapses, ideally, into the shape of a regular hexagon—however, considering the thickness of poster-board and the slight imperfections of a complex construction magnified, you might have to apply some pressure to accomplish this.

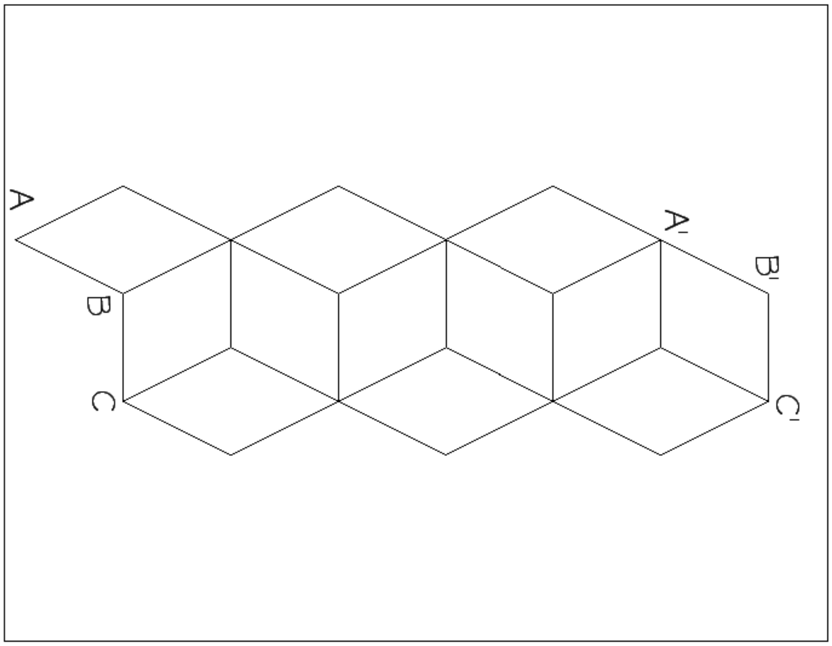


Figure 11.6

5. Another way of folding the same construction results in a Polar collapsoid, which folds up around a polar axis by joining the edges marked by arrows as shown in **Figure 11.7**. Ideally, this 12-celled polar collapsoid folds flat into the shape of 4/6ths of a regular hexagon. Her drawings of the finished models (**Figures 11.8 and 11.9**) are used here with her permission.

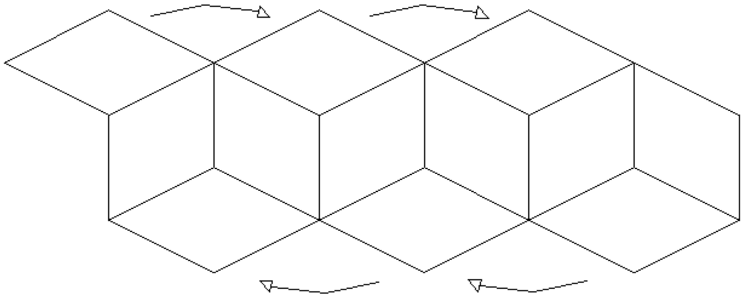


Figure 11.7

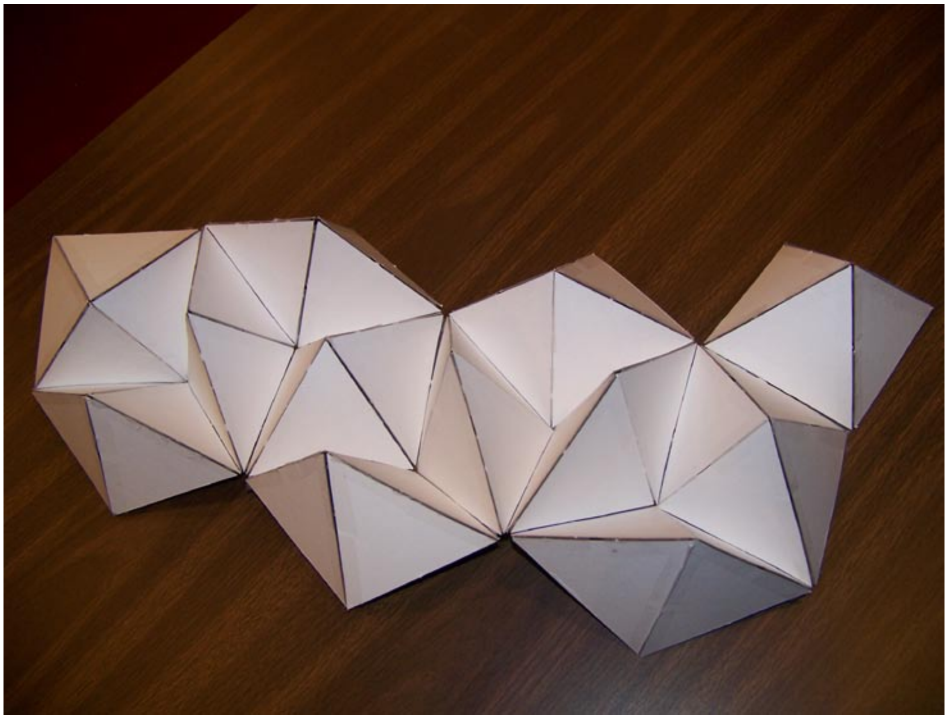


Figure 11.8

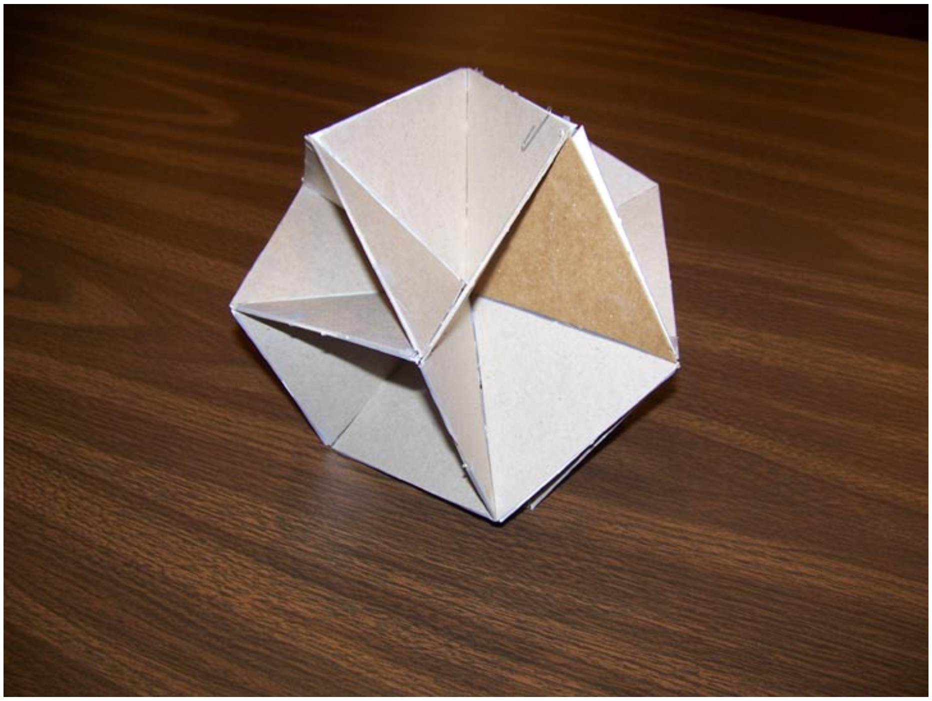


Figure 11.9

Professor Pederson's 30-celled Collapsoid, the Pseudo-Triacontahedron

Let's move on to a somewhat more complicated model, the 30-cell collapsoid, which Prof. Pedersen identifies as a "pseudo-triacontahedron." This model, she points out, is linked to the icosahedron and the dodecahedron, both of which have 30 sides.

1. For the equatorial version of this collapsoid, we must make 30 baseless four-sided pyramids, this time using an Isosceles triangle derived from a decagon. To obtain this shape, simply create a pentagon as shown in Chapter 5, and use **Figure 5.10** to create a decagon from a pentagon. Or, you can even use this pattern to design a template (**Figure 11.10**). Use this template to make all 30 cells of poster-board. Score, cut out, and fold the pieces into 30 baseless pyramids. Assembly will take some patience!

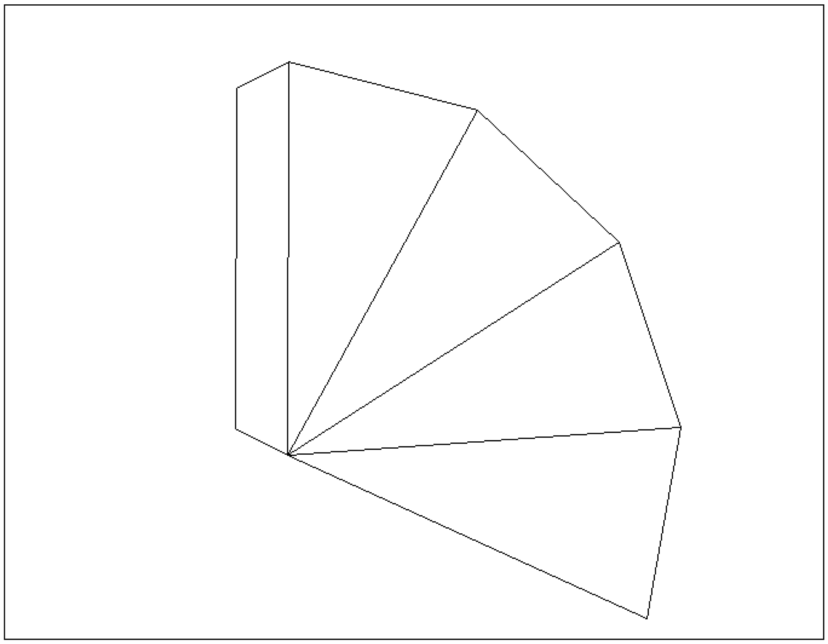


Figure 11.10

2. Arrange ten of these pyramid cells in a row, taping them edge to edge just as we did in **Figure 11.4**, along their common edges. Turned up-side-down, this assembly will look like ten tall pyramids in a row. Add another cell, #11, in the angle between cells 1 and 2, connecting one edge to the first cell side, and another edge to the second. Introduce another cell between 3 and 4, then 5 and 6, and so on until cells 11 through 15 have been added. In the angle formed between cells 11 and 12, add another cell, No. 16), doing the same between 12 and 13, 13 and 14, 14 and 15. A final cell along this side (no. 20), attaches to cell 15 by one common edge only. See **Figure 11.11**.

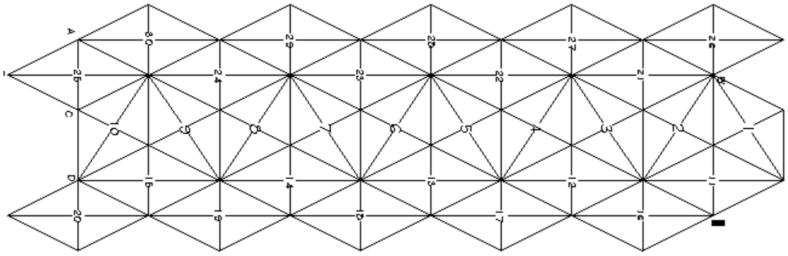


Figure 11.11

3. Follow the same process on the other side of the series of ten, introducing cells 21, 22, 23, and 24 in the angles formed by cells 2 and 3, 4 and 5, 6 and 7, 8 and 9. A final cell, no. 25, connects to 10 by one common edge only. Five cells remain to be attached,

the first, no. 26, by one face only to the upper edge of cell 21, and the remainder, 27, 28, 29 and 30, in the angles between cells 21 and 22, 22 and 23, 23 and 24, and 24 and 25.

4. The finished assembly takes on dimensional form by joining edges AB to A'B', BC to B'C', CD to C'D', and DE to D'E' and holding them together with bits of tape. The completed version is shown in **Figure 11.12.**

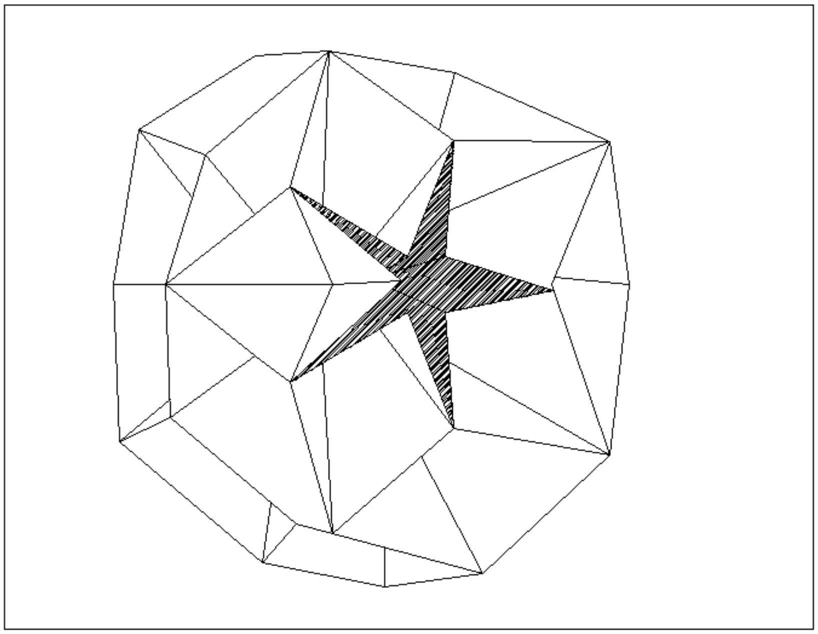


Figure 11.12

5. A functional Polar version requires the use of a pyramidal base made of equilateral triangles, derived from a hexagon. If you wish to attempt it (as if you haven't had enough with this version!), follow the same assembly instructions as above, using the net given in **Figure 11.13**, and the connections indicated in this case by arrows. In the case of this Polar collapsoid, if the tape-tabs are released, it should fold flat into the shape of a hexagon (with lots of pressure!)

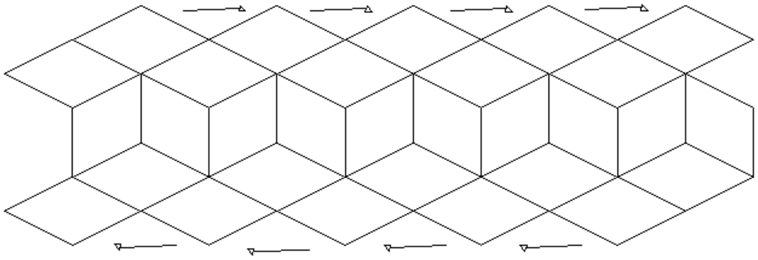


Figure 11.13

Professor Pederson's 20-celled Collapsoid, the Pseudo-Rhombic Icosahedron

1. Prof. Pedersen provides another interesting collapsoid, this one based on 20 cells. For this one, however, the preliminary Isosceles triangle is based on the octagon—actually, half an octagon. To determine this, simply inscribe a square in a circle, then, bisect each of its four sides. Draw diagonals. Connect consecutive points on the circle to create the octagon. See **Figure 11.14**. **Figure 11.15** can be used as a template for half the octagon.

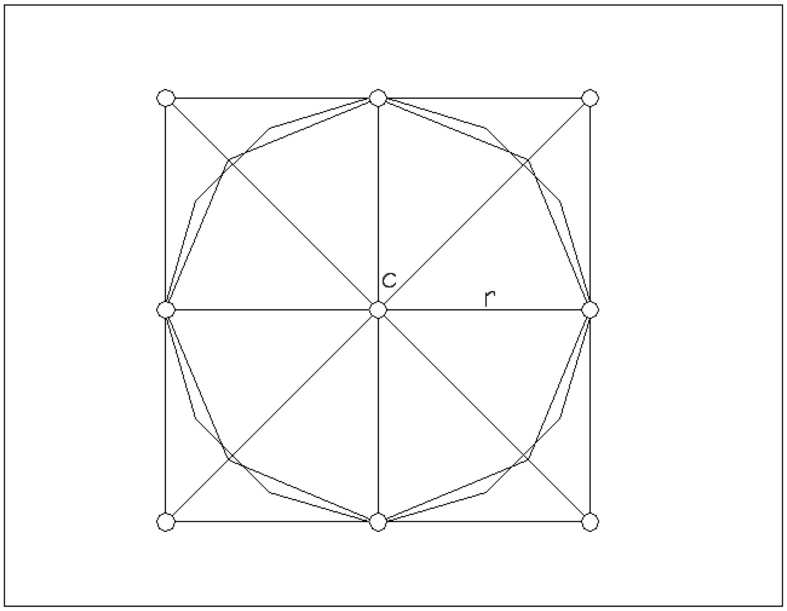


Figure 11.14

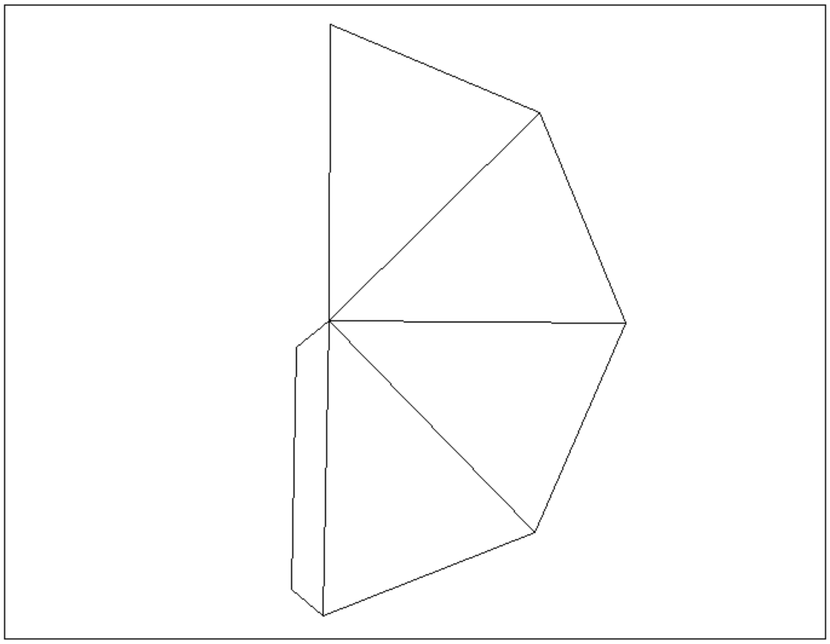


Figure 11.15

2. The pattern given in **Figure 11.16** resembles a slightly more complex version of the previous 12-cell construction, but be careful! There is a tricky step here. Connect cells 1 through 8 by their common edges in a long row. But note, in our illustration, how the “saw blade” pattern is thrown off at cell 5, before correcting itself for the remaining cells, 6, 7, and 8. This allows for the addition of two “extra” cells: 9 and 10. Cell no. 11 connects along one edge only to cell no. 1, while cell #12 shares two edges with cells 2 and 3. Cell 13 shares one edge with cell #4, and the other with cell 9, while the last cell on this side, 14, shares edges with 9 and 10.

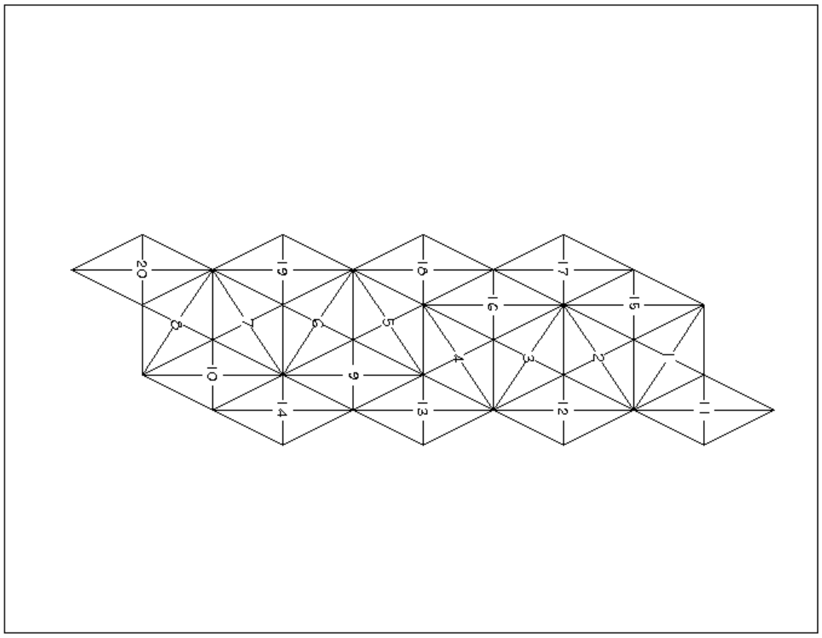


Figure 11.16

3. Along the opposite side, cell 15 joins cells 1 and 2, and cell 16 joins cells 3 and 4. The last four cells are attached in the following fashion: cell 17 is attached to 15 and 16, 18 to 16 and 5, 19 to 6 and 7, and 20 by a single edge to 8. Then, when all this work is done, you can create the equatorial form of this collapsoid by joining edge AB to A’B’, BC to B’C’, and CD to C’D’.

4. The polar form uses the same net, but the cells in this case must be made of equilateral triangles derived from the hexagon, and then uniting the edges marked by arrows as in **Figure 11.17**. I would be remiss if I did not include the name of this construction, which Prof. Pedersen calls a “pseudo-rhombic icosahedron.”

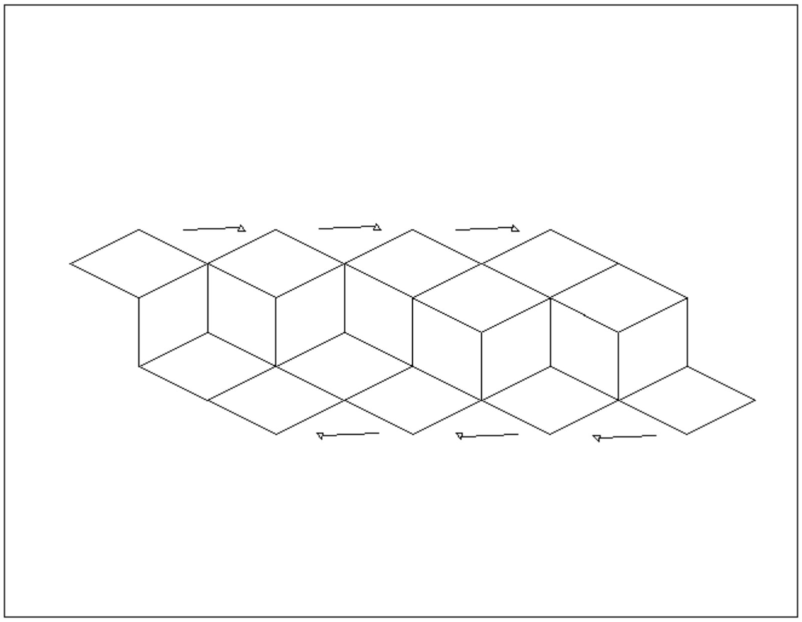


Figure 11.17

5. In every instance, following the net as carefully as possible will produce the desired result. Instead of poster-board, a heavy stock paper might increase the possibility of actually “flattening” your collapsoid constructions, but the sheer bulk and density of the construction and the accumulation of slight imperfections might work against you.

An Additional Exercise:

While there are other, more complex collapsoid possibilities, all are based on the process described thus far, and while interesting to scholars, involve a considerable amount of repetition in the construction of the models. If the topic fascinates you, I refer you to Dr. Pedersen’s article on collapsoids in **Mathematical Gazette.**

Giant versions of the first two collapsoids are especially rewarding after all the hard work—especially the first, which is great fun to play with.