

## CHAPTER 3 - LET'S LEARN A LITTLE GEOMETRY!

### Making the Perfect Triangle

Much has been written about the symbolism of the triangle, specifically the equilateral triangle, and there can be no doubt that it is the simplest and purest of all polygons. If the circle symbolically represents God because it has no beginning and no end, the equilateral triangle stands as a symbol of the Trinity, or triune God of the Christian faith. If you superimpose two equilateral triangles on top of each other, one pointing upward and the other downward, you have the Jewish Magen David, or Shield of David. Among the various interpretations, this symbol is said to represent the universal authority of God in all directions. Little wonder, then, that the triangle has been revered over the centuries as a kind of magical symbol. It's versatility in any number of geometric applications reinforces that reputation. As you will see in this chapter, the links between the circle and the equilateral triangle are many.

#### **Objective:**

To create a perfect equilateral triangle

#### **Vocabulary:**

Adjacent  
Circle  
Circumference  
Decahedron  
Deltahedra  
Diameter  
Dipyramid  
Equidistant  
Equilateral Triangle  
Hexagon  
Icosahedron  
Net  
Octahedron  
Pentagon  
Platonic Solid  
Polygon  
Polyhedron  
Pyramid  
Radius  
Tetrahedron

First of all, you need to create a circle, or at least parts of a circle, in order to make a perfect equilateral triangle. Draw a circle with a compass, and without changing the setting of your compass, you can mark off three points on the circle which make a

perfect equilateral triangle if connected by straight lines. Inscribe another equilateral triangle using the spaces between the points, and you come up with a Magen David: six small equilateral triangles around a hexagon center, which itself can be subdivided into six equilateral triangles! The same formula works to subdivide a circle into six large equilateral triangles, like cutting up a pie. There just doesn't seem to be an end to the wonderful creations you can make with the equilateral triangle, as we will demonstrate in this chapter. Not only will we produce three of the five Platonic Solids, you can make a number of odd constructions that will fit perfectly well here.

### Exercise 3

#### Materials needed:

Standard white paper

Several large sheets of 22 x 29-inch white poster-board (cut in half to the more manageable dimensions of 22 x 14 1/2-inches),

A straight-edge

Sharp No. 2 pencils

A quality compass

A pair of scissors

Cellophane tape

An eraser

1/2-inch x 3 1/2-inch white label strips (recommended)

#### Steps:

#### The Equilateral Triangle

1. To make a fast and accurate equilateral triangle, you don't even have to start with a line. A single point which we will name Point A on a piece of paper (or on the blackboard) is enough to get you started. Set your compass to the size of triangle you want, put the compass point on Point A and sweep out a wide arc. Mark another point, Point B on the arc and sweep another arc from point A to intersect your arc at point C. These three points are all equidistant from each other, and when connected by lines, form a perfect equilateral triangle. **(Figure 3.1)**

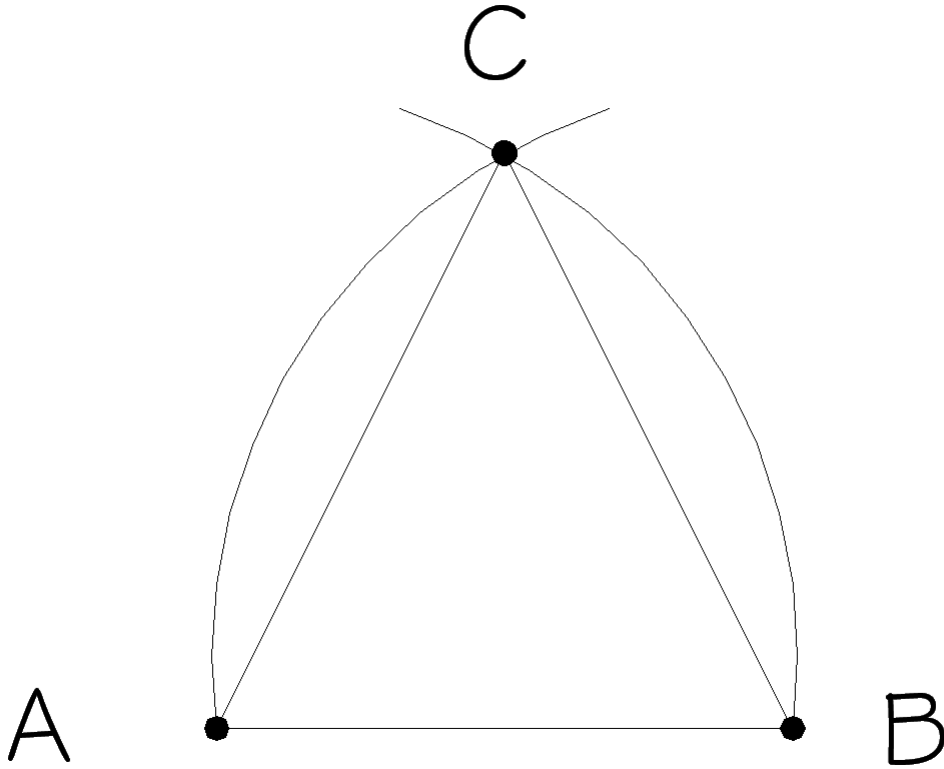
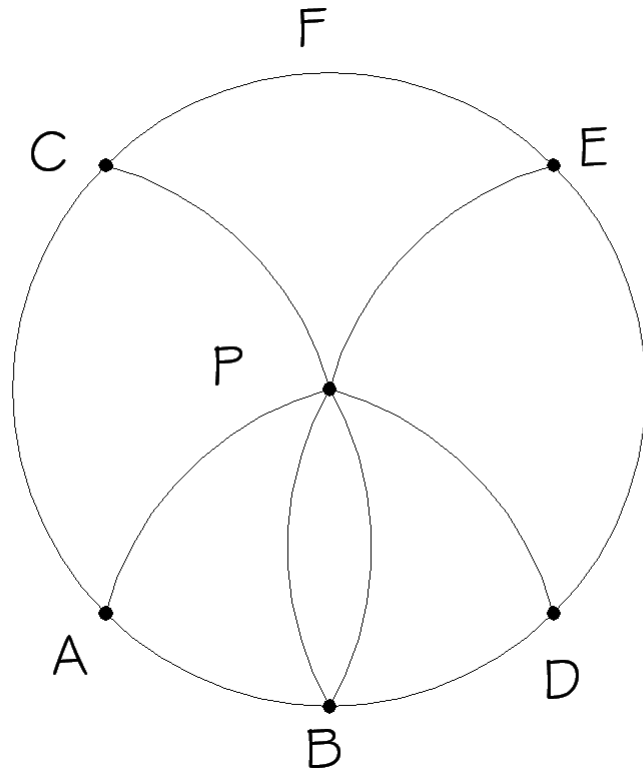


Figure 3.1

### The Circle Flower

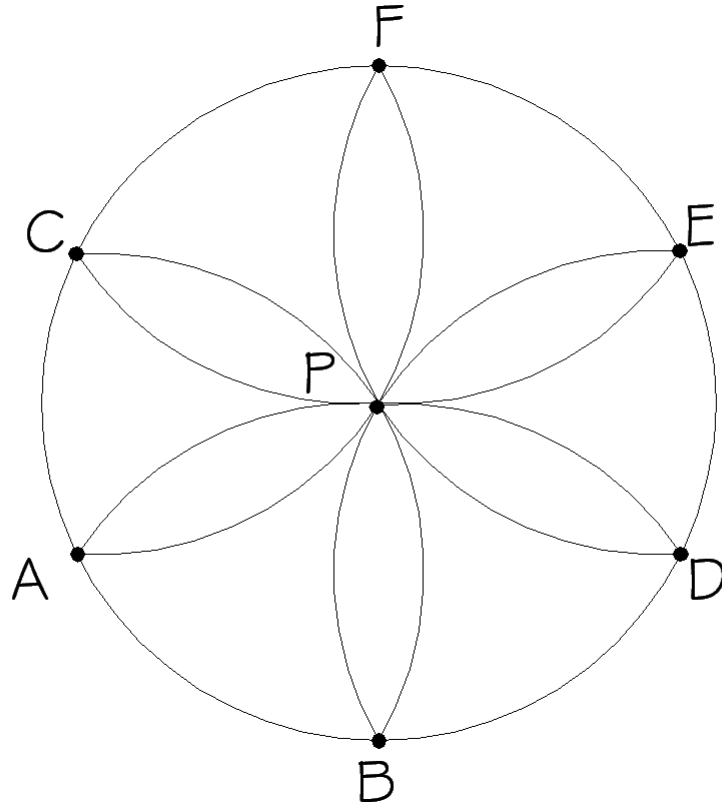
An interesting exercise in creating big and small triangles begins with what I call the, “Circle Flower”. This drawing must be accurate, and therefore requires some patience and skill. The slightest deviation in your compass at any time during the process will produce poor results. But there are ways to check for errors along the way, and even if we are not completely successful the first time around, we can always begin again.

1. Set your compass to a radius of four inches, and place the compass point at the center of your paper. Call the center Point P. Make a full sweep around to create a circle with a diameter of 8 inches. Once you’ve made the circle, set the compass point anywhere on the circle (**Figure 3.2, Point A**) and sweep an arc across the center of the circle from point B to point C on the circumference. Now set the compass point to B and do the same without changing the radius, sweeping a curve from A to D. Do the same at D, sweeping an arc from E to B.



**Figure 3.2**

2. Continue this process all around the circle, until you have a “flower” of six petals. Every arc should pass through Point P, center of the circle, and end at petal tips around the circumference of the circle. This is the Circle Flower. **(Figure 3.3)**



**Figure 3.3**

3. Draw line segments connecting the tips of the flower petals directly across from each other, passing through Point P in the center. Then connect consecutive points around the circumference of the circle. You have now drawn a hexagon, or six-sided polygon, made up of 6 equilateral triangles. (**Figure 3.4**)

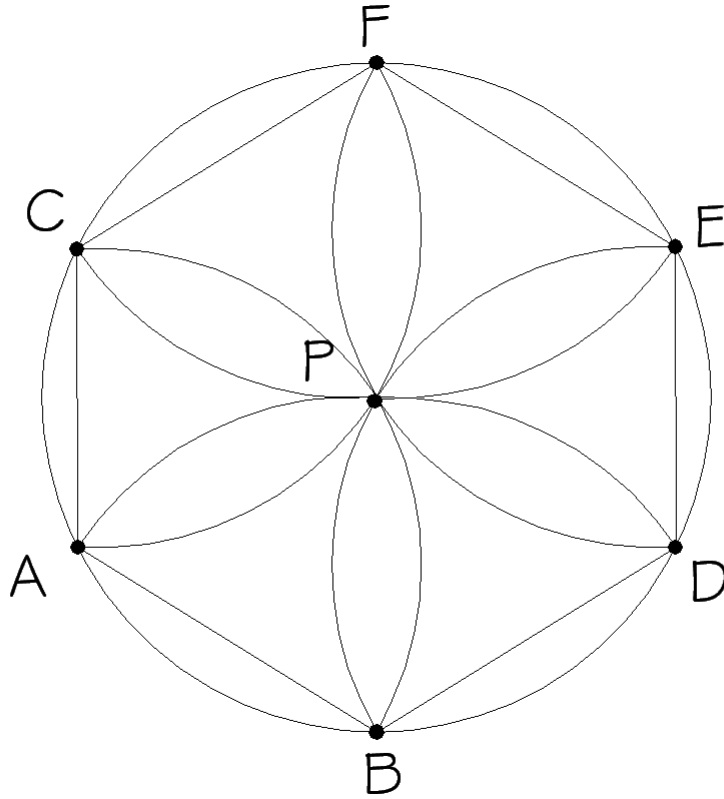


Figure 3.4

### The Star of David

1. Refer to Figure 3.3 to complete the following" exercise. Erase the petals, leaving the points created at the tips. Connect alternate points on the edge of the circle, A, D, and F, to produce one large equilateral triangle. If you connect the three remaining alternate points C, B, and E to form a second large triangle on top of the first, the result is a perfect six-pointed star, known as the Star of David. Observe that 6 additional equilateral triangles are formed around a hexagonal center. (Figure 3.5)

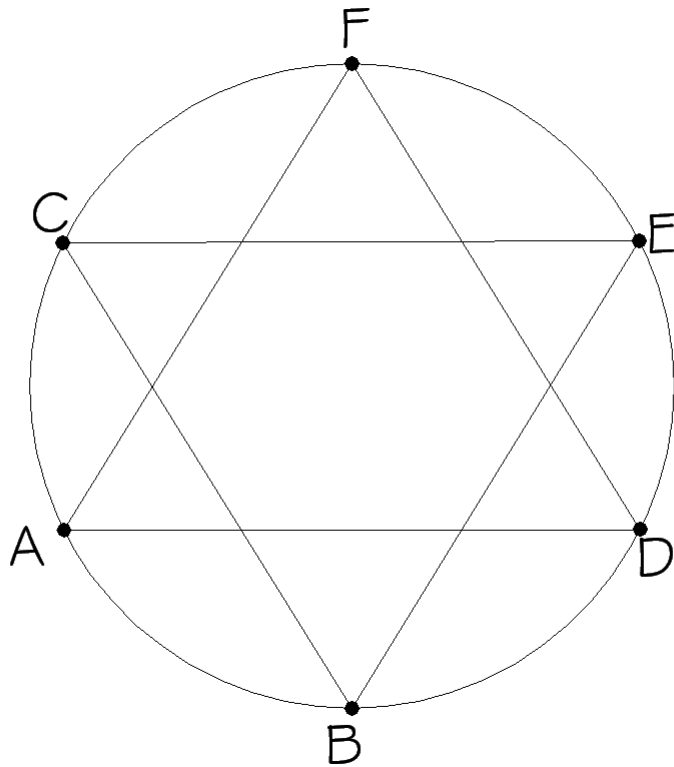
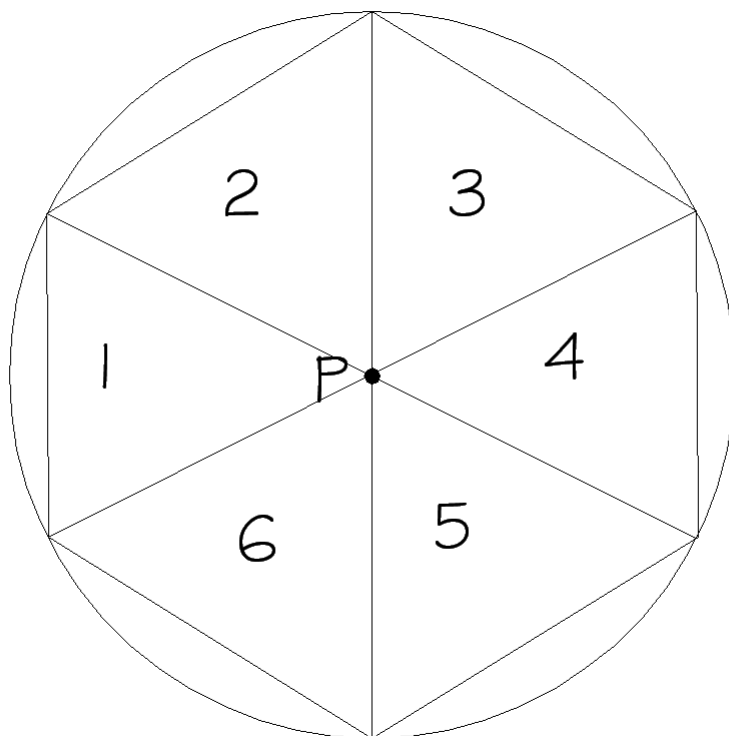


Figure 3.5

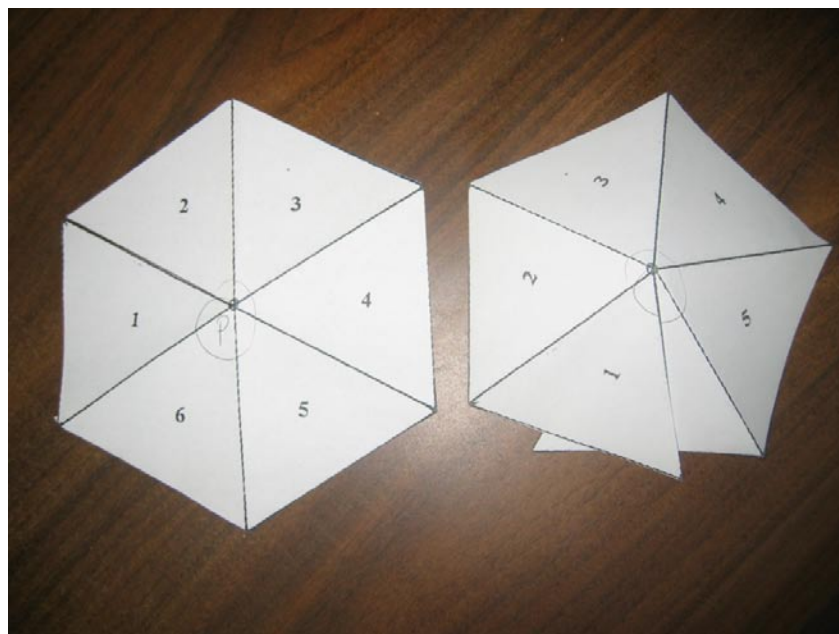
### The Decahedron

1. Refer to Figure 3.4 to complete the following exercise. Transfer Figure 3.4 to poster board without drawing in the petals. Once you know how to make the “flower,” you don’t have to reproduce it in its entirety in order to locate the points needed to lay out the hexagon or the triangles. You can use the compass to score points A, C, D, E, F, and B. Draw in the lines to make the hexagon, and then criss-cross the hexagon with three diameter lines to divide it into 6 equilateral triangles. (Figure 3.6) Then cut out the hexagon created within the circle.

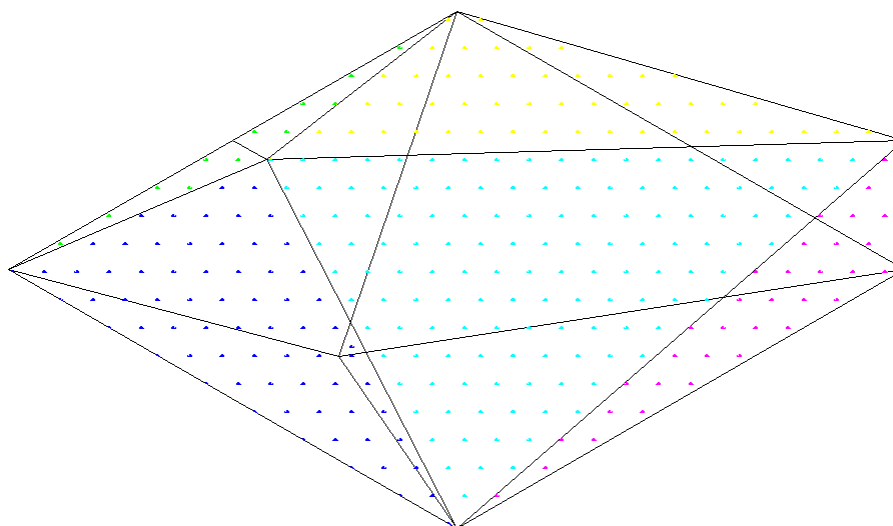


**Figure 3.6**

2. Now, the fun begins. Cut into the hexagon from one of the points on the circumference to the center, Point P. Gently fold each of the triangles along the lines, and pull one triangle completely under the triangle adjacent to it. This will cause the construction to “bulge” slightly forming a very shallow five-sided pyramid or a shell. **(Figure 3.7a)** The open base will take the outline of a perfect pentagon, or five-sided polygon. Apply a small piece of tape just to hold this arrangement together--not a permanent bond. A second “shell” made exactly like this one and connected base to base will form a ten-sided shape called a “decahedron” or ten-sided polyhedron, better known as a pentagonal dipyramid. **(Figure 3.7b)**



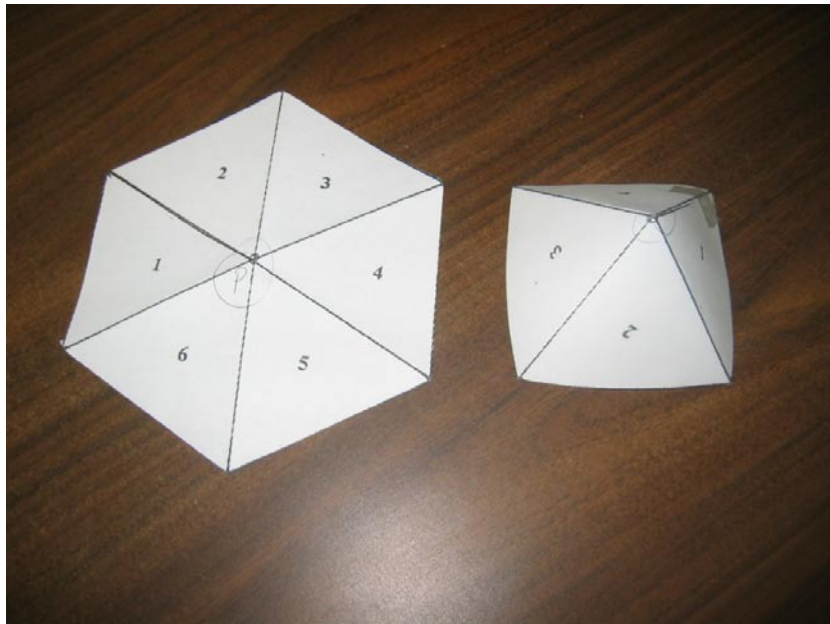
**Figure 3.7a**



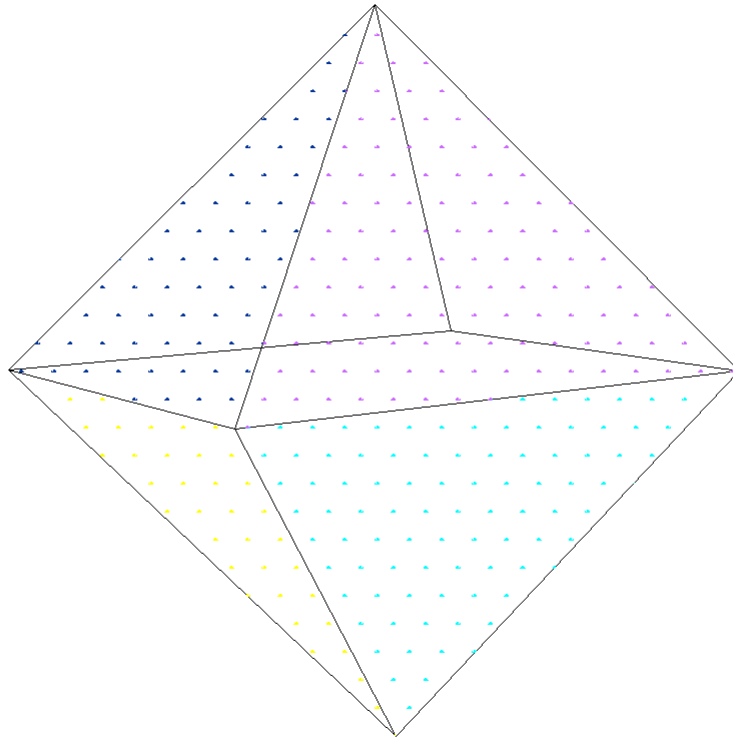
**Figure 3.7b**

## Octahedron

3. Next, pull the tape loose on this construction, and overlap another triangle, so that 2 triangles are completely under another 2 triangles. Use a little tab of tape to secure the new shape. This will now give the appearance of a traditional four-sided “Egyptian” pyramid with an open base forming a perfect square. (**Figure 3.8a**) Once again, two such pyramids placed base to base and taped together results in a beautiful solid called an octahedron (Eight-sided), another one of the five Platonic Solids (**Figure 3.8b**) Why is this a Platonic Solid and the pentagonal di-pyramid is not? Refer to the rules defining a Platonic Solid, and see how it applies to the octahedron, but not to the pentagonal di-pyramid. The rules are - all faces have exactly the same shape, all edges are of equal length, and all vertices (corners) are of equal angle measure.



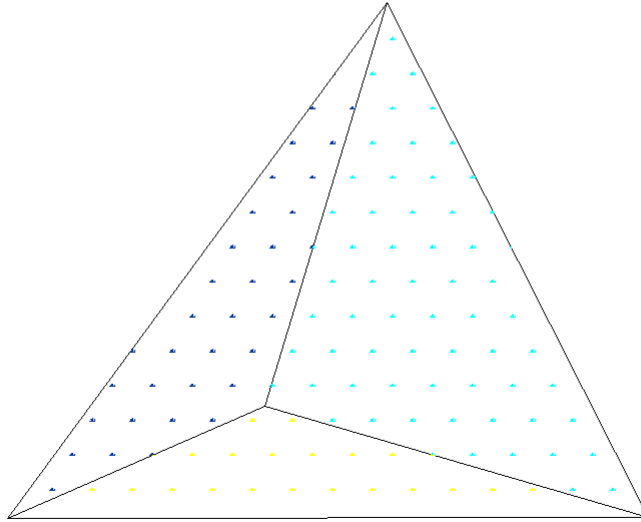
**Figure 3.8a**



**Figure 3.8b**

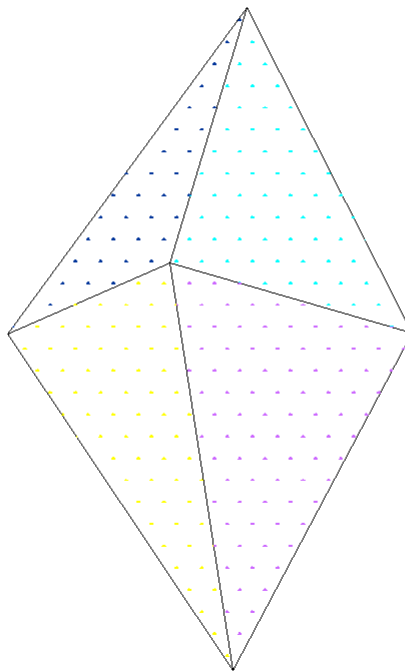
### **Tetrahedron**

4. Now, overlap three triangles to create a three-sided pyramid, with an open base in the form of an equilateral triangle. If you enclose the base, the construction would become a tetrahedron (four-sided), yet another of the Platonic Solids. (**Figures 3.9a**)



**Figure 3.9a**

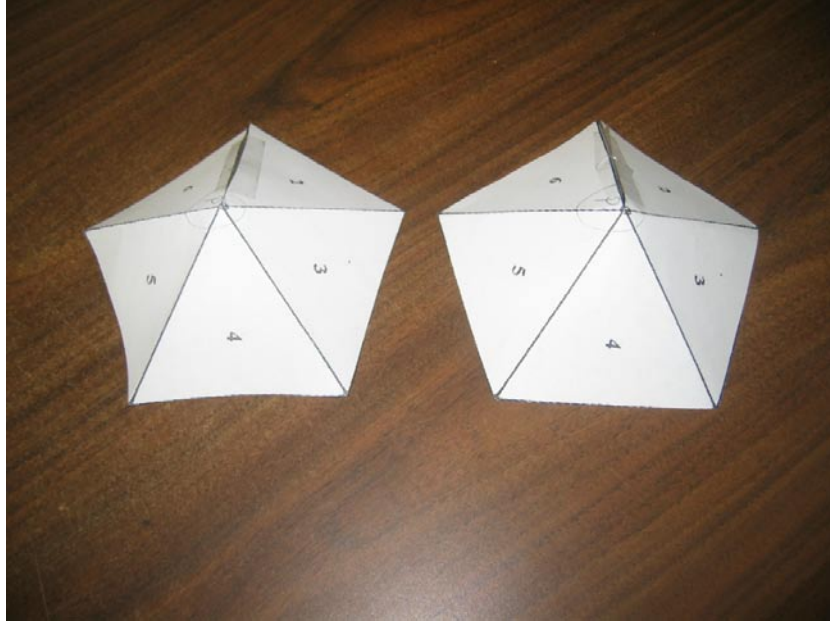
5. By putting two of these three-sided pyramids together base to base, the resultant form would be a triangular dipyramid. It is also known as a hexahedron (six-sided) just like the cube, but it is not a Platonic Solid. **(Figure 3.9b)**



**Figure 3.9b**

### **Icosahedron**

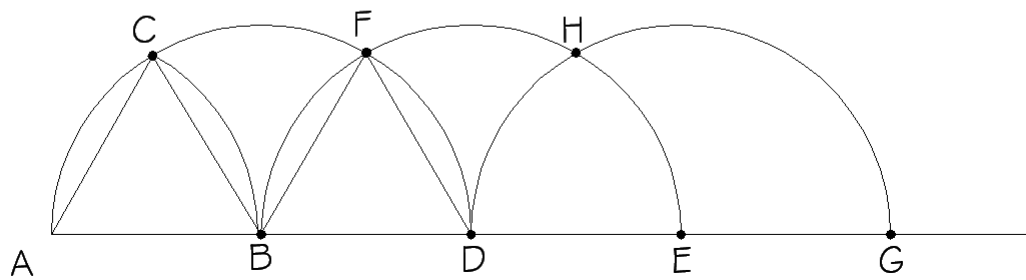
6. One more exercise remains to be completed with our “shell.” Open the shell to expose all six triangles, and use this form as a pattern to make another exactly like it. When this second hexagon has been created, cut out one of the triangles completely from both hexagons. Bond the edge of one triangle to the edge of the adjacent triangle with a full-length strip of tape to create two shells. Set both “shells” aside. (**Figure 3.10**)



**Figure 3.10**

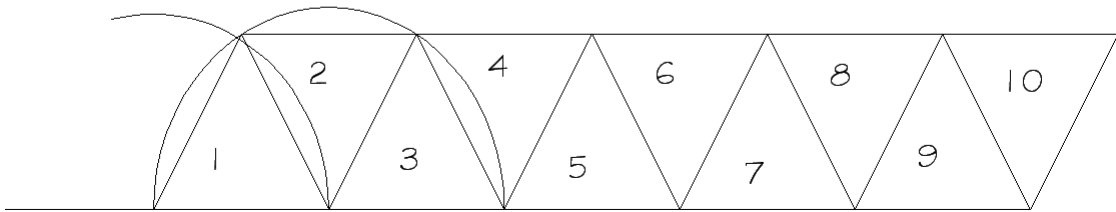
7. What we want to make now is a row of ten equilateral triangles, of exactly the same size as those in our former construction. This can be easily and rapidly done by drawing a straight line on a piece of poster-board. If your original triangles are four inches on a side, this line should be at least 20 inches long. This length will accommodate all ten triangles needed to complete the construction.

8. Place a point at the left end of your line and label it, Point A. Set your compass to four inches, and place the compass point on Point A. Draw a wide arc crossing the line and label the point created, Point B. Now, with the compass set on point B, draw an arc crossing the line and label Points C and D as illustrated in the diagram, **Figure 3.11**. Draw a line from Point C to Points A and B to create an equilateral triangle. Nine more triangles to go! Set the compass point at D and spin a wide arc from B to E. This arc will intersect the previous arc at F, giving you the three points needed to create another triangle. Continue this process until you have created five equilateral triangles.



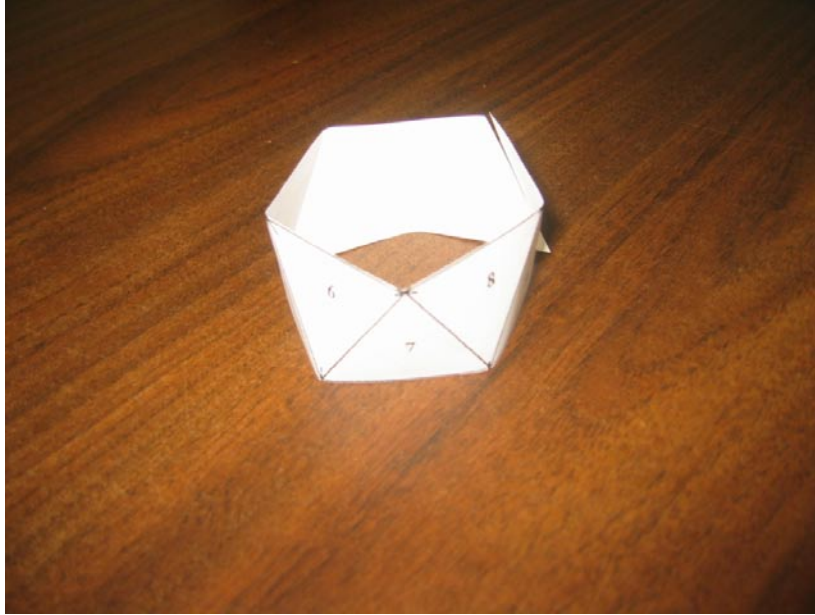
**Figure 3.11**

9. Study your work carefully, and you will note that if you connect the top vertices of this series of triangles by drawing a straight line you can create a series of ten equilateral triangles. (**Figure 3.12**)



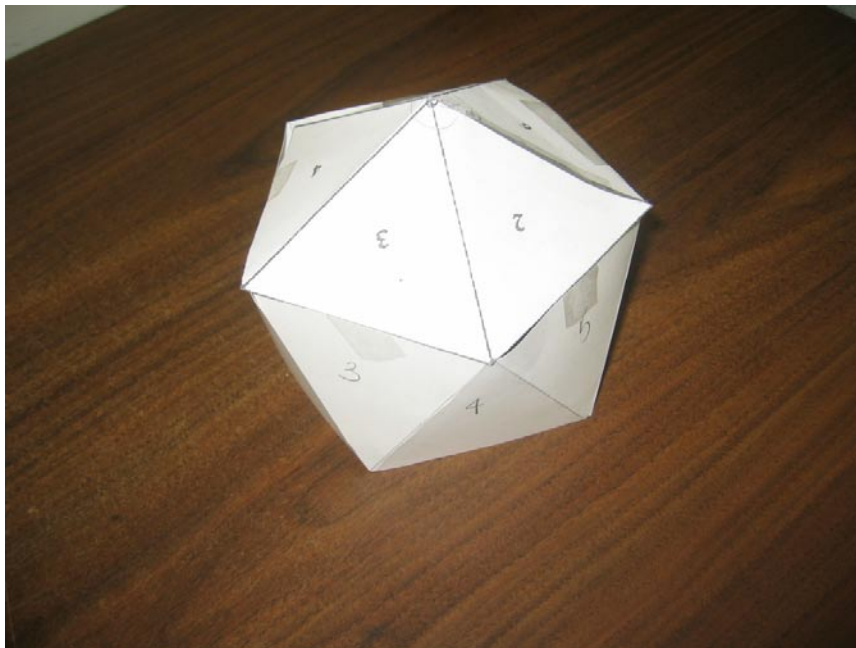
**Figure 3.12**

10. Make sure that you have ten triangles, then cut them out as a single long strip without separating the triangles. Fold along the edges of the triangles to make a loop, and tape the edge of triangle 1 to the edge of triangle 10. Notice that the open bases of this new structure form pentagons. (**Figure 3.13**)



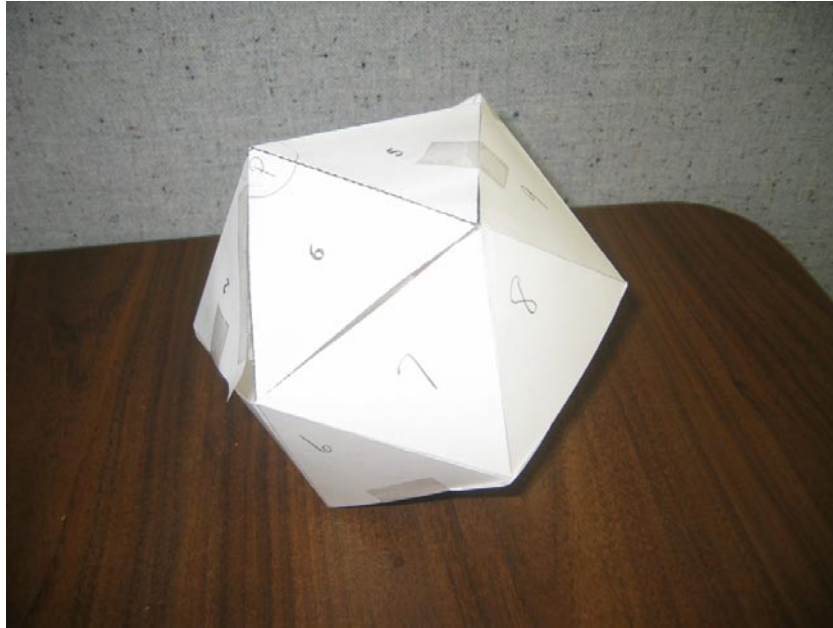
**Figure 3.13**

11. Now take one of the “shells” that you made earlier, set it on the top of this pentagonal base, aligning the common edges, and tape these edges together. If you have been careful to maintain the same dimensions of all your triangles, the “shell” should fit perfectly on the band of triangles like the lid on a pot. It will look a little like a hat. Try it on! (**Figure 3.14**)



**Figure 3.14**

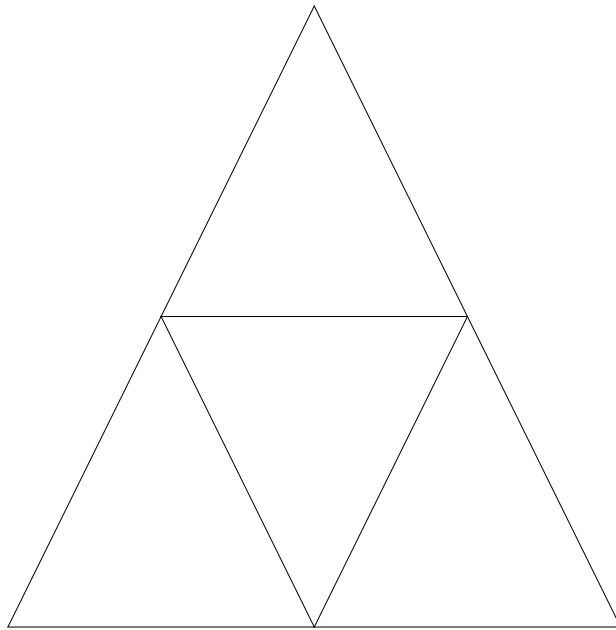
12. Affix the second “shell” to the bottom of the ring of triangles with strips of tape to enclose the construction. The finished product is a platonic solid known as the Icosahedron (20 sides). (**Figure 3.15**)



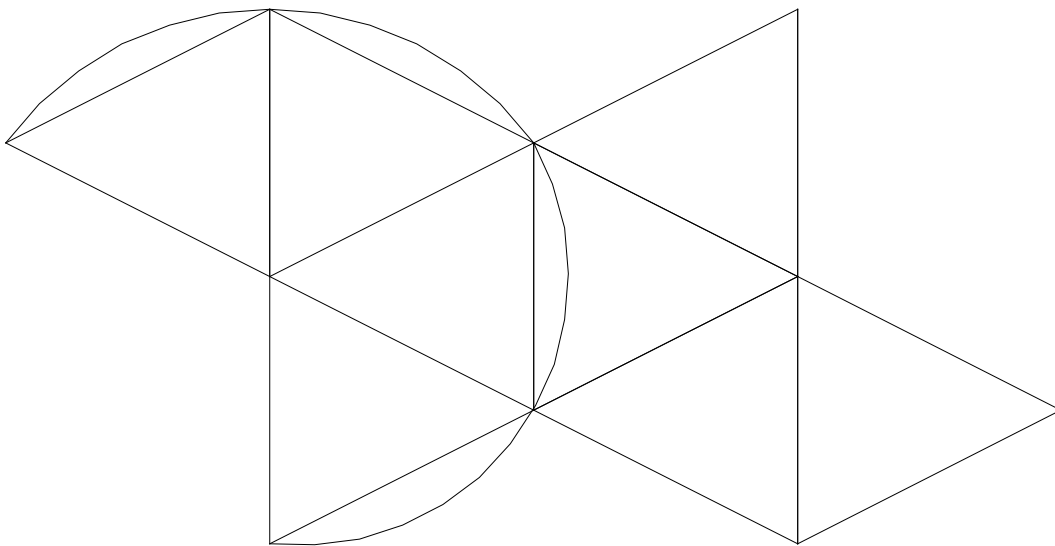
**Figure 3.15**

***Additional Exercises:***

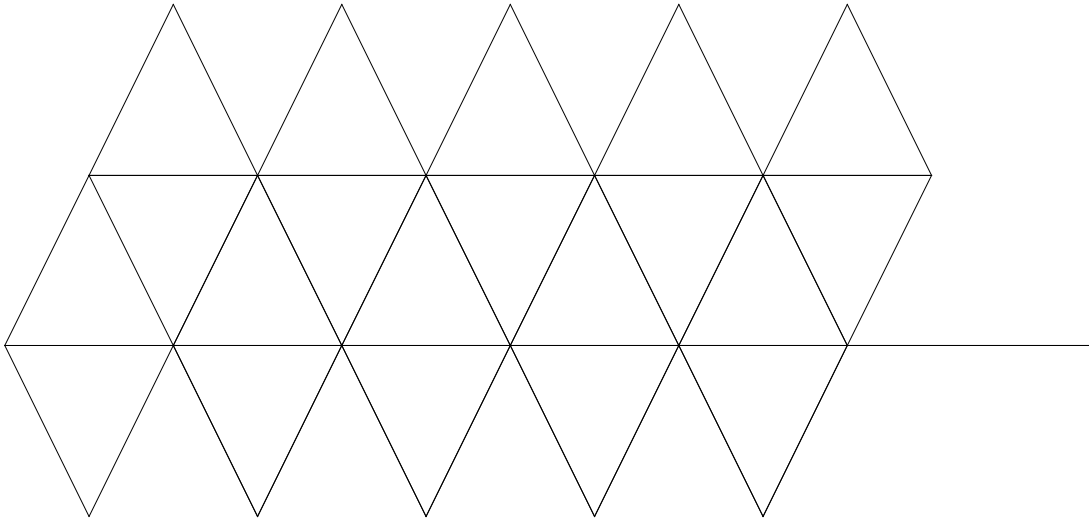
1. If you are interested in making an individual tetrahedron, octahedron, or icosahedron without going through the steps outlined in this chapter, the nets for all three are shown in **Figures 3.16, 3.17, and 3.18**. \*Note: All scoring shown must be made with a compass opened to a consistent radius length.



**Figure 3.16**



**Figure 3.17**



**Figure 3.18**

2. You have probably taken note of the fact that three of the Platonic Solids are made up of equilateral triangles. Actually, there are a number of polyhedrons you can make using nothing but triangles. The smallest has four (a Platonic Solid), then six, eight (a Platonic Solid), ten, twelve, fourteen, sixteen, twenty (a Platonic Solid), and even sixty sides. All such triangular constructions are called deltahedra, and all consist of an even number of equilateral triangles. Isn't it curious how this series grows by even numbers, yet skips 18?