

CHAPTER 5 - THE PENTAGON AND THE DODECAHEDRON

The hardest design of all is the pentagon, but with patience, you can master this technique. After the simplicity of the equilateral triangle and the square, and their resultant Platonic Solids, the pentagon comes as something of a challenge. To make a perfect five-sided pentagon requires following a complicated process, and the utmost accuracy with the best tools. It is a real credit to Euclid that he had solved the problem of this polygon so long ago, and done it so accurately. There are several approaches to designing a pentagon, but the formula presented in this chapter is the best.

In ancient times, the pentagon was a symbol associated with the Goddess Venus, emphasizing her war-like aspect. Some ancient fortresses take this shape, and we need only remind ourselves of our own Pentagon in Washington DC where military decisions are made every day. The pentagon is related to the pentagram, the star symbol we find in symbolic use all over the world. Note its use on our own American flag. If you draw line segments from each vertex of a pentagon to all non-adjacent vertices, a star inside the pentagon will result. Likewise, if you take a pentagon and extend each of its five lines to a certain distance, they will meet at five points to create a star, or pentagram. Cut this design out of poster board and fold the star points together, using the enclosed pentagon as a base, you will have the first of the twelve stellations needed to make a stellated dodecahedron.

Objective:

Learn how to create a three-dimensional pentagon and dodecahedron.

Vocabulary:

Arc
Bisect
Chord
Compass
Contiguous
Decagon
Dodecahedron
Golden Section
Inscribe
Pentagon
Pentagram
Perpendicular
Radius

Materials:

Standard white paper

Several large sheets of 22 x 29-inch white poster-board (cut in half to the more manageable dimensions of 22 x 14 1/2-inches),

A straight-edge

Sharp No. 2 pencils

A quality compass

A pair of scissors

Cellophane tape

An eraser

1/2-inch x 3 1/2-inch white label strips (recommended)

Steps:

The Pentagon

1. Draw a horizontal line lengthwise on a sheet of paper using a straight edge. Then draw a circle around point X near the center of the line using a compass. Label the intersecting points on the circle, A and B.
2. Construct a perpendicular line using the directions in **Chapter 4**. Then label the intersecting points on the circle, C and D. You have now divided the circle into 4 equal quarters.
3. Divide the radius AX by constructing a perpendicular. Set your compass point on point A and open it to point X. Sweep arcs directly above and below the radius, line segment AX. Then, place the point of the compass on point X and repeat. Draw a line to connect the points of intersection to form the perpendicular at point G. Point G is the midpoint of radius AX. (**Figure 5.1**)

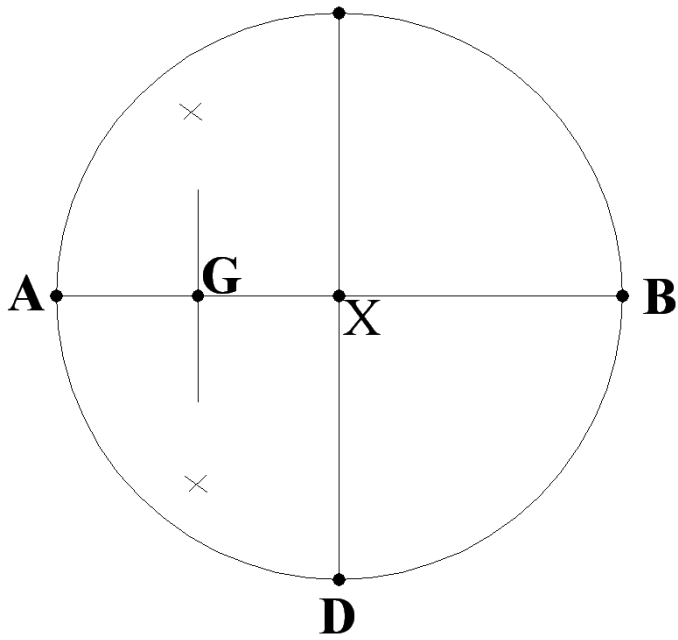


Figure 5.1

4. Set your compass point at G. and set the pencil tip at C, and sweep an arc down to radius XB. Label the point of intersection, H. The distance between point C and point H is the length you need to inscribe a perfect pentagon within the circle. (Figure 5.2)

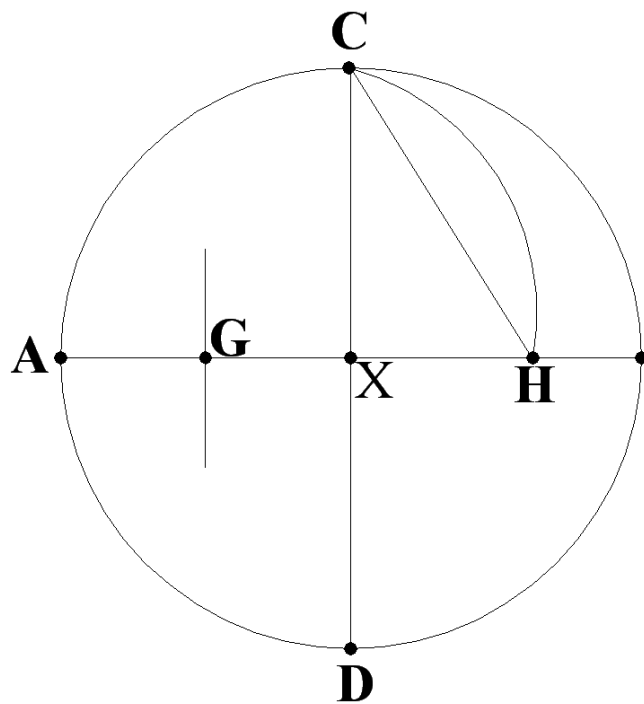


Figure 5.2

5. To do this, set the compass point at C with the pencil tip at H, and sweep a small arc out to the circle. Label the point of intersection, I. Draw a straight line from C to I. (Figure 5.3)

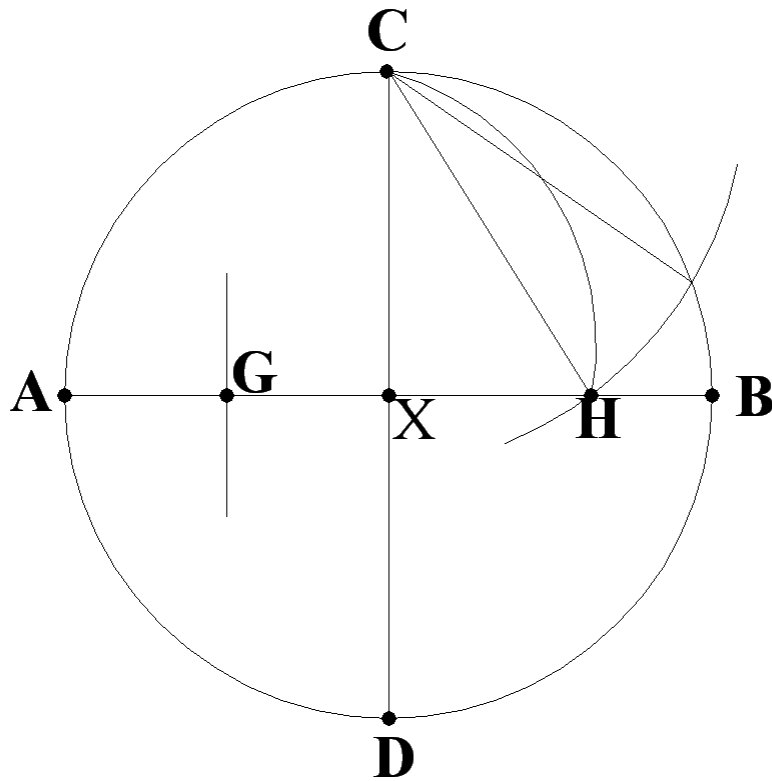


Figure 5.3

Chord CI is one-fifth of a circle, and by repeating this chord around the circle with your compass at points C, I, J, K, and L, you can complete your pentagon. (Figure 5.4)

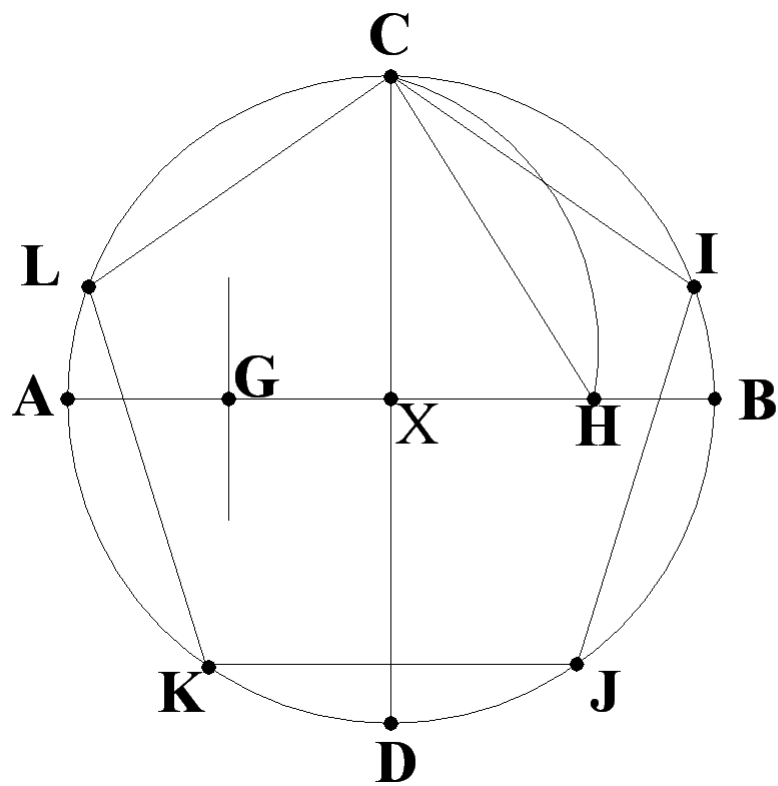


Figure 5.4

The Dodecahedron

1. The pentagon you created can be used as a template or lay out the pattern for the dodecahedron. This is shown in **Figure 5.5**.

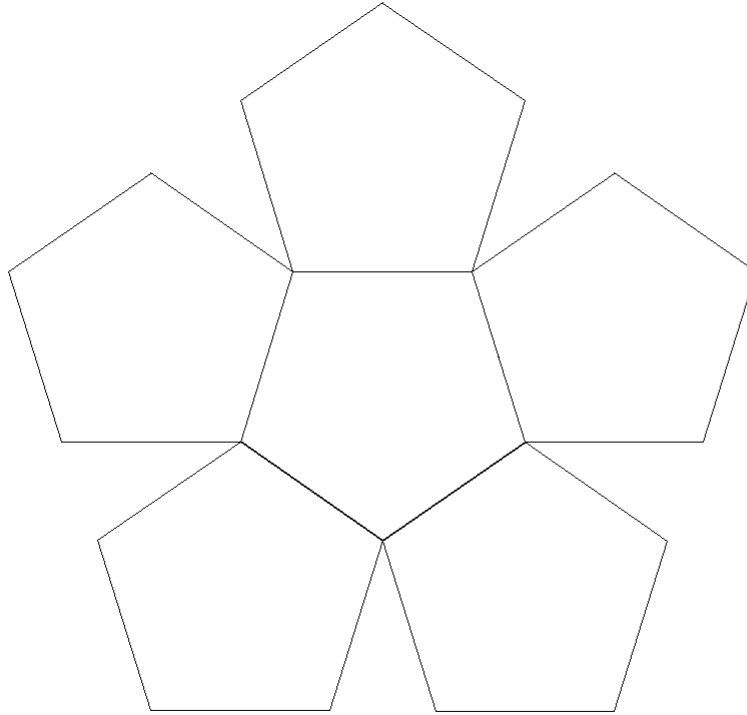


Figure 5.5

2. Score the inside lines, and cut out the net. Lift two adjacent outer pieces until the edges come together, and carefully tape them in place. Do the same all around until you have formed a basket-like shell. Go over the outside edges with tape to reinforce the structure. Repeat the process to make the other half. (**Figure 5.6**)

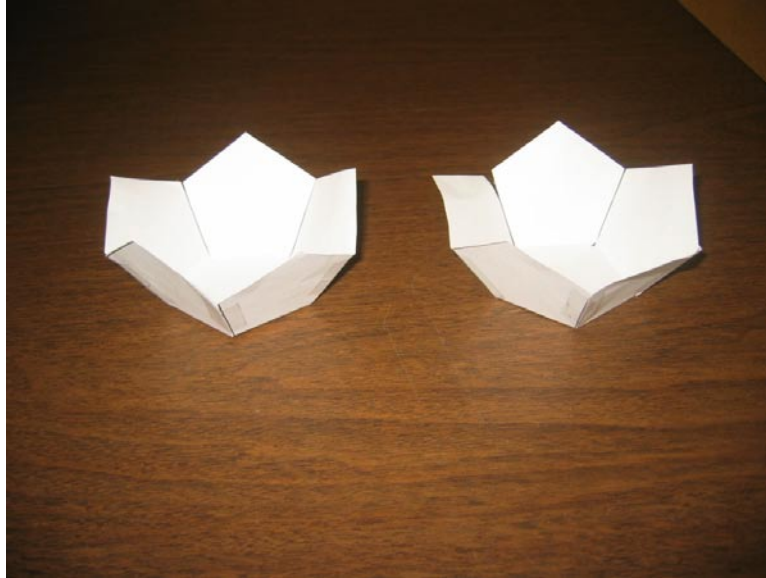


Figure 5.6

3. The two halves will fit together neatly, but taping these together may take some doing. Tape one set of common edges at a time, and work your way around the solid until all ten common edges are sealed. (**Figure 5.7**)



Figure 5.7

Additional Exercises:

1. The method given above to design a pentagon does not allow you to create one to a pre-determined size. If you want to make a pentagon of a specific size, begin by making one as described above without regard to dimensions. When this pentagon is complete, use your straight-edge to extend or shorten two line segments (sides) of a common angle to the length you desire. See **Figure 5.8**, line segments AB and BC.

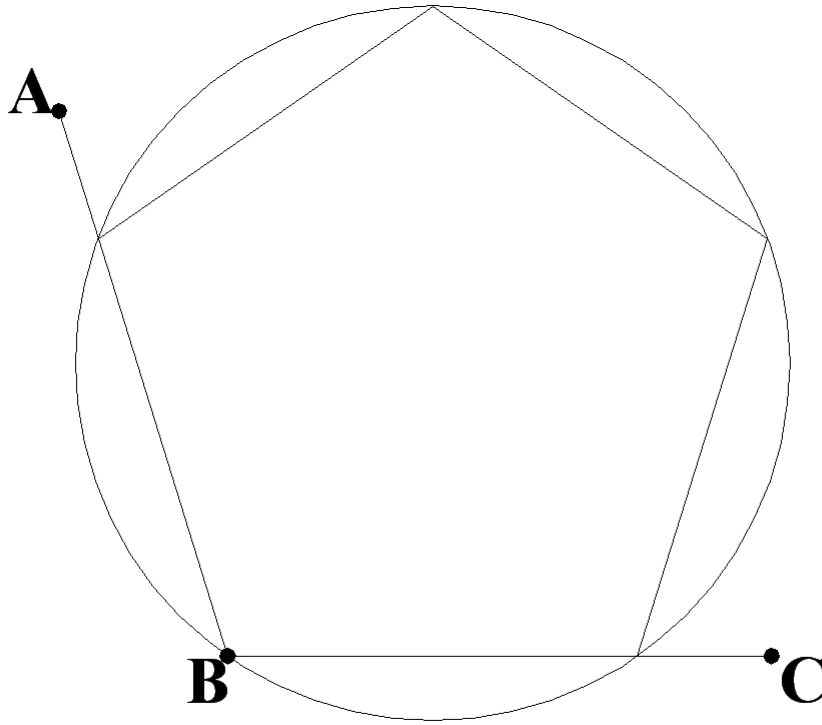


Figure 5.8

2. Using the end-points of lines AB and BC as reference points, construct perpendiculars to the center of each line. Extend the perpendiculars to the inside of the angle until they intersect at point Z. Using point Z as the center of the “new” circle, spin a new circle with your compass set to radius ZA. This will enclose line segments AB and BC as chords. Reset your compass to the length of line segment AB, and mark off the additional points needed on the new circle to complete your properly sized pentagon. **(Figure 5.9)** *This same process can be used later on to enlarge the octagon and the decagon.

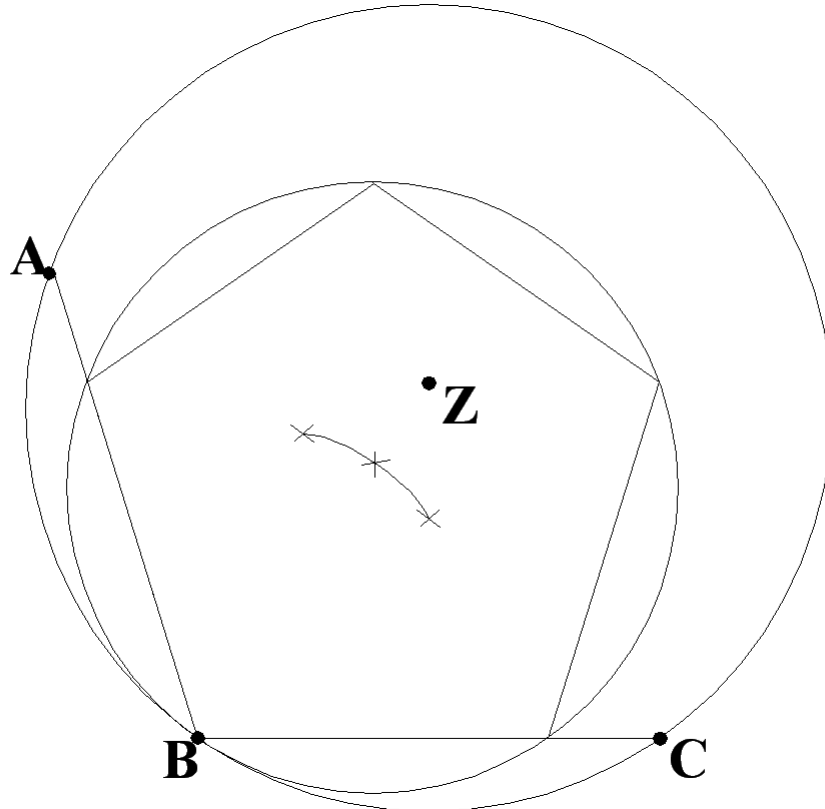


Figure 5.9

2. Eventually, it will be necessary for you to know how to produce a decagon, ten-sided polygon, so the technique is provided in **Figure 5.10**. Construct a pentagon in a circle in the usual way, and then bisect each side of the pentagon. Lines carried from the center of the circle through the midpoint of each bisected line to the arc of the circle will result in 10 chords, or a perfect decagon.

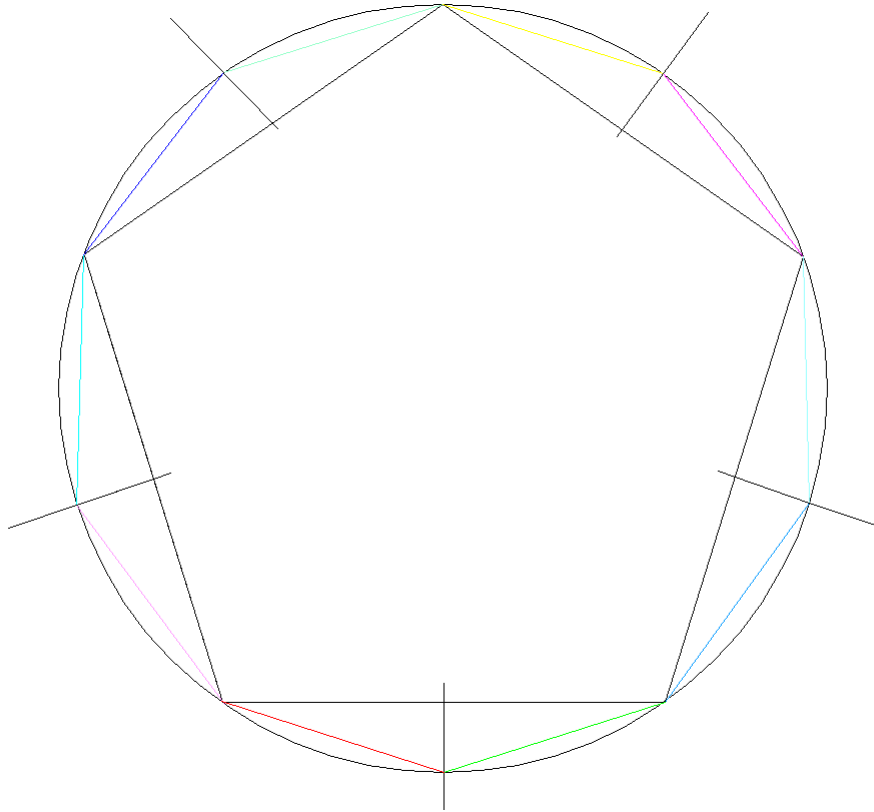


Figure 5.10

3. You will also find it interesting to note that lines drawn from each interior angle of a pentagon to a non-adjacent angle will result in a pentagram, or a five-pointed star. **(Figure 5.11)** In like manner, extending all five lines of a pentagon will similarly result in—an even larger five-pointed star!

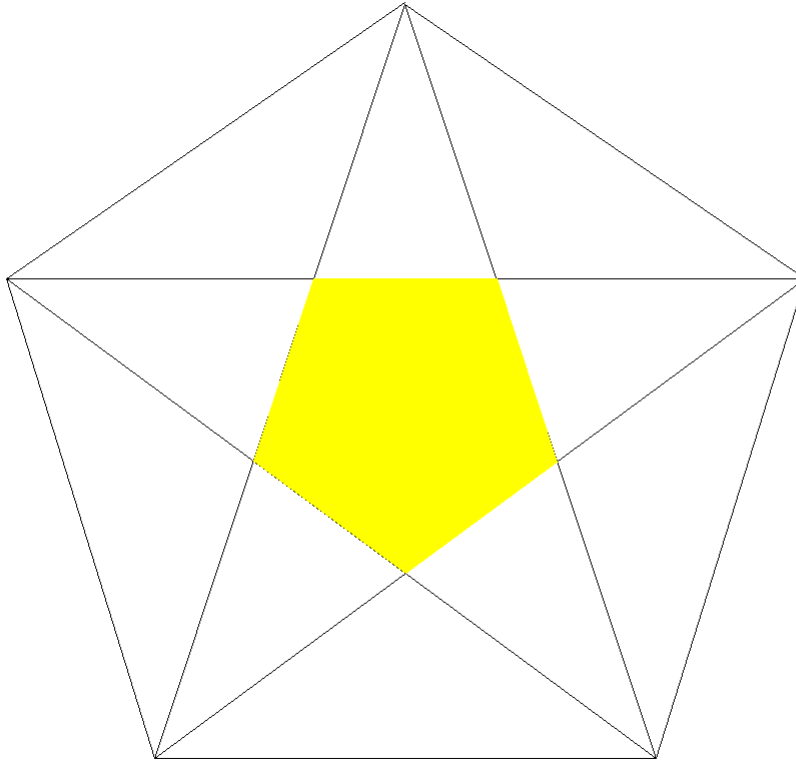


Figure 5.11

4. It is strongly recommended that when making basic shapes like the square, the equilateral triangle, and the pentagon, especially those you may wish to keep as templates (patterns), they all be made using the same basic dimensions. In some future exercises, we will be making different types of solids in which different shapes are combined (squares and triangles; squares, triangles, and pentagons; hexagons and squares, etc.), and these templates will prove to be quite handy.