CHAPTER 9 - THE ARCHIMEDEAN SOLIDS

In chapters 3, 4, and 5, we surveyed the complete set of the five Platonic Solids (Chapter 3 – Octahedron, Tetrahedron, and Icosahedron, Chapter 4 – Cube, Chapter 5 – Dodecahedron). The continuation of this series is a larger group known as the Archimedean Solids, so-called because they appear to have been known to the Greek mathematician and inventor, Archimedes, in the 3rd century BC. There are thirteen solids in the Archimedean series, and as you might expect, they become increasingly more complex and larger in size.

The Platonic Solids are all "regular" solids, because all faces are from polygons of the very same shape. The Archimedean Solids are all "semi-regular" solids, because they are made of combinations of the same polygons we have used before--the equilateral triangle, the square, and the pentagon--with the addition of three new shapes: the hexagon, the octagon, and the decagon. Eleven of these solids are a combination of two polygons, while the remaining two solids use three different polygons in their construction. Despite this, the vertices (corner angles) of any Archimedean Solid are all equal to each other.

The first five Archimedean Solids bear a close resemblance to the five Platonic Solids because they are "truncated" versions of these. "Truncated" simply means "cut off." If we were to cut off the corners of every Platonic Solid (carefully measured, of course), we would be left with the first five Archimedean Solids.

Objective:
To create all 13 three-dimensional Archimedean solids.

Vocabulary:
Archimedes
Archimedean Solids
Platonic Solids

Materials:
Standard white paper
Several large sheets of 22 x 29-inch white poster-board (cut in half to the more manageable dimensions of 22 x 14 1/2-inches),
A ruler
Sharp No. 2 pencils
A quality compass
A pair of scissors
Cellophane tape
An eraser
1/2-inch x 3 1/2-inch white label strips (recommended)

Instructions for Assembly

In most cases, merely studying the "net" and reconstructing it on poster-board will provide all the guidance needed to assemble the Archimedean solids. To the extent that you can, construct these from the inside with clear tape, making sure that the tape is pressed completely and snugly into the angles between the two connecting edges. In this way, all your "accidents" with the taping will be hidden away. When you can no longer reach inside, you should take as much care as possible in applying the tape, for an attractive finished appearance.

In earlier sections, we recommended that you observe a specific scale in making all the pieces for the five Platonic Solids, and that you save a "template" or pattern of each one for later applications. You should therefore have an equilateral triangle, a square, and a pentagon, all with uniform edges. To complete the Archimedean series, three new polygons will be needed, one which you already know how to design (the hexagon), and two others yet to be introduced (the octagon and the decagon). ***Pre-packaged kits of all the Platonic Solids and most of the Archimedean Solids, as well as most of the other models discussed in these exercises can be purchased from The Canton Museum of Art, at (330) 453-7666, all of which come complete with tape and assembly instructions.**

A Note on Building "Giant" Solids

Most of the Archimedean Solids (as well as the Platonic Solids) are particularly attractive and quite fascinating if constructed on a larger scale. While it may be a laborious process, cutting quantities of cardboard polygons in assorted shapes using a one-foot module will produce solids ranging in size from the tiny Tetrahedron to the gigantic rhombitruncated icosidodecahedron. Such huge construction projects generate much excitement with students of all ages, the younger ones enjoying the added thrill of actually climbing inside! Such large constructions are ideal for display, decoration, or even for a museum-sized exhibition.

Exercises

1. Truncated Tetrahedron

1. First construct the Tetrahedron, **Figure 9.1**. See Chapter 3, Tetrahedron.

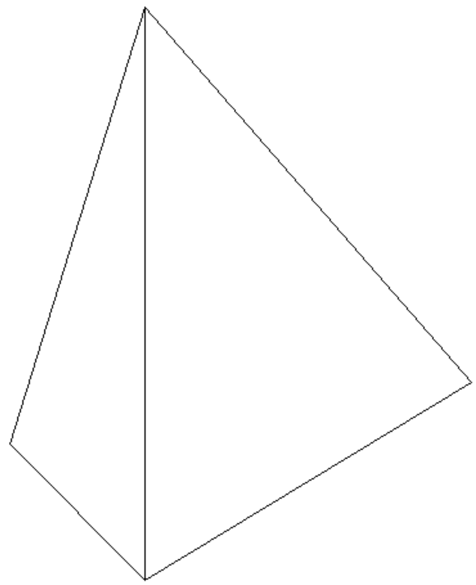


Figure 9.1

2. Cutting off each of the 4 tips results in a “truncated tetrahedron.” In fact, the “cut-off” portion turns out to be a miniature tetrahedron itself, and each face of the “cut” is a perfect equilateral triangle. By cutting off the tips of the tetrahedron to create its truncated version, you introduce a new polygon-shape on each of 4 faces: the hexagon (**Figure 9.2**).

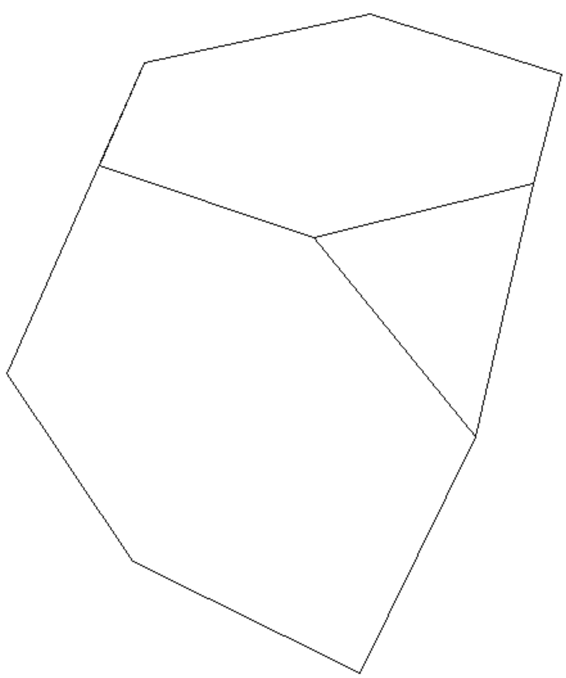


Figure 9.2

Of course, we are not actually going to cut up a tetrahedron. Rather, we will design and make this solid in its truncated version from the start. This solid has four hexagon faces and four equilateral triangle surfaces. If you make the truncated version to the same scale as your original tetrahedron (the edge of the triangle is equal to the edge of the hexagon), you will find that the two fit together perfectly.

2. In **Chapter 5**, we introduced the hexagon and showed how to make one by means of the "Circle Flower." Unless you are using a template to copy the pattern, follow this process again until you have made a hexagon.

3. Make three more in a row, connected by a common edge, as in **Figure 9.3.**

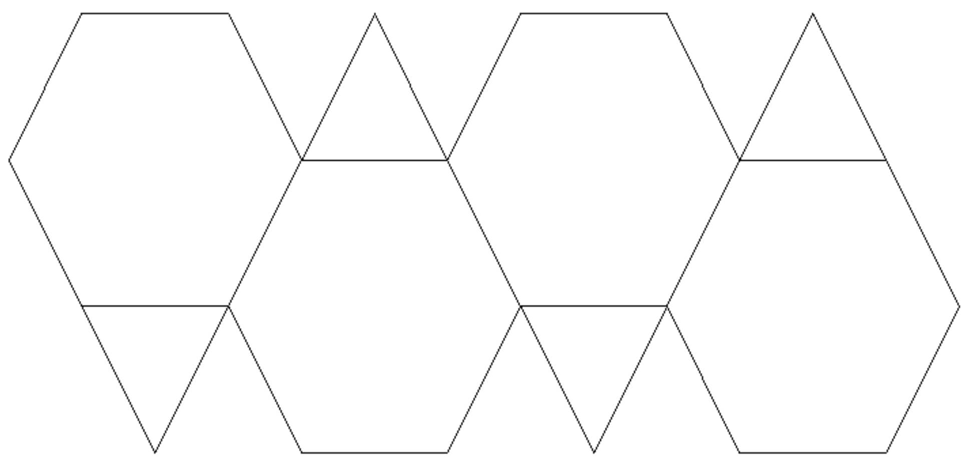


Figure 9.3

4. Add one equilateral triangle tip to each as shown. (Review **Chapter 3** for the construction of an equilateral triangle. Score all the inside lines and cut out. Your form will quickly take shape. **Figure 9.4** shows the completed truncated tetrahedron.

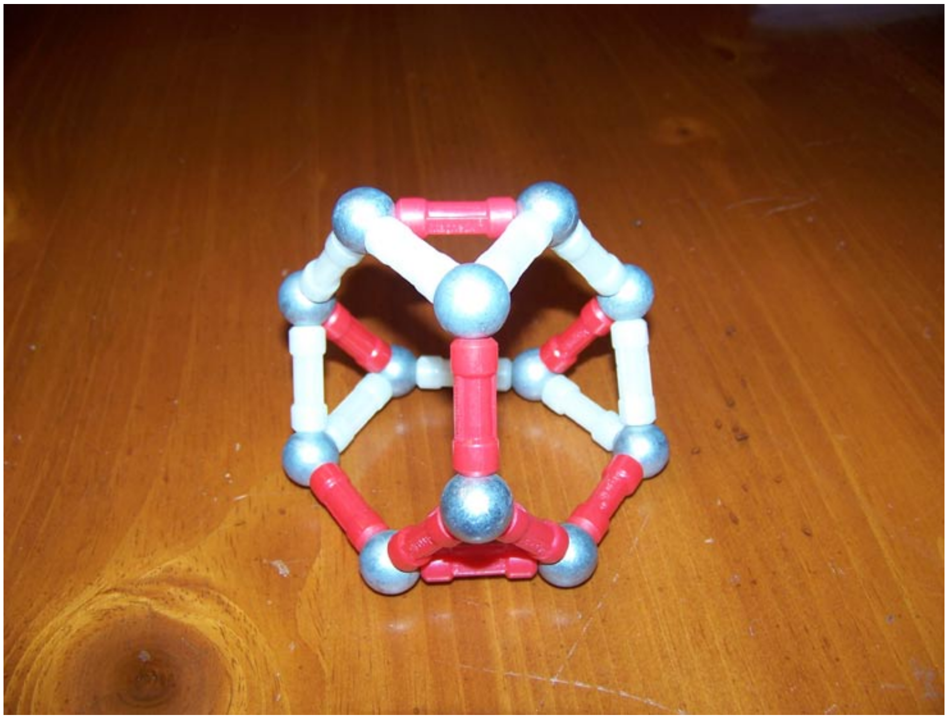


Figure 9.4

2. Truncated Octahedron

1. In similar fashion, cutting off the 8 vertices of the octahedron (**Figure 9.5**) will result in its truncated version, the truncated octahedron (**Figure 9.6**).

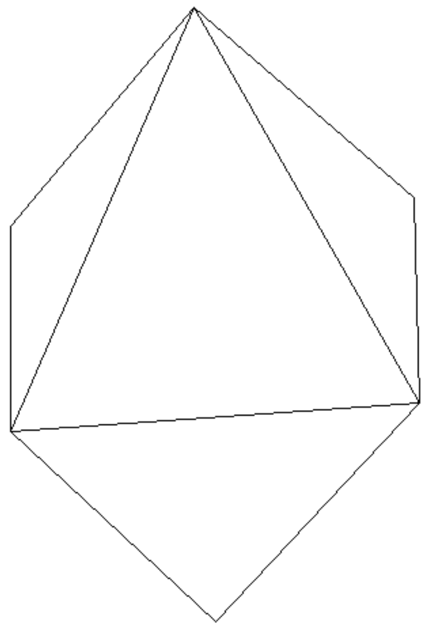


Figure 9.5

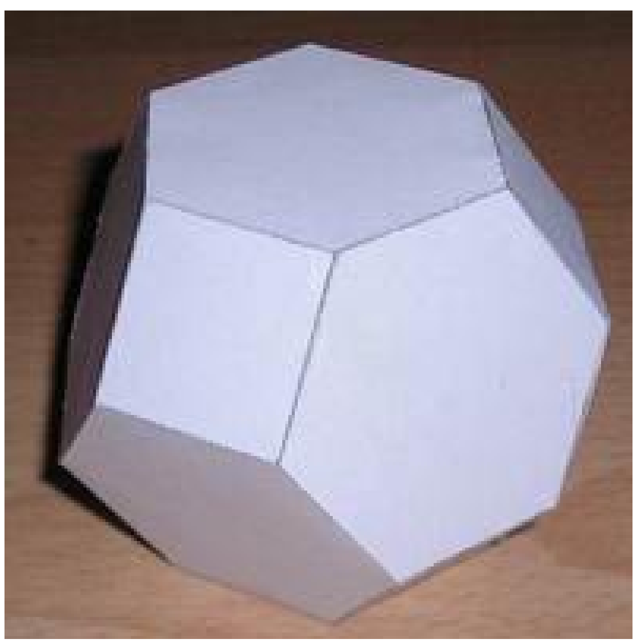


Figure 9.6

2. You can see from the original solid, final assembly, and net, that the cutting off process results in a semi-regular solid with eight hexagon faces and six square faces. Putting a tetrahedron on any of these truncated faces will not re-create the original octahedron shape. Think for a moment--what shape will you need to complete the six cut-off corners of your truncated octahedron? Follow the net provided (**Figure 9.7**) or use the directions in **<u>Chapter 3</u>** to create all the hexagons and **<u>Chapter 4</u>** to create all the squares to produce a small version.

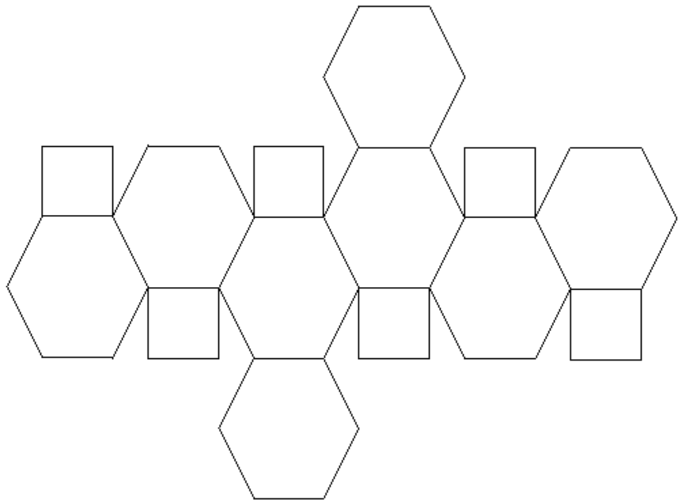


Figure 9.7

The simplest approach, for a larger version, is to lay out one hexagon piece, and surround it with alternating squares and hexagons along the common edges, as we have done in **Figure 9.8**. Score, cut out and assemble. This will result in a bowl-like shape, which is actually <u>half of the form</u>. Set this aside, and make the other half just like the first, except allow one hexagon flap to move freely--don't tape either side. This way, you will have access to the interior of the shape for taping until the very last. The hexagon flap itself will have to be taped from the outside. The completed construction is shown on **Figure 9.9**.

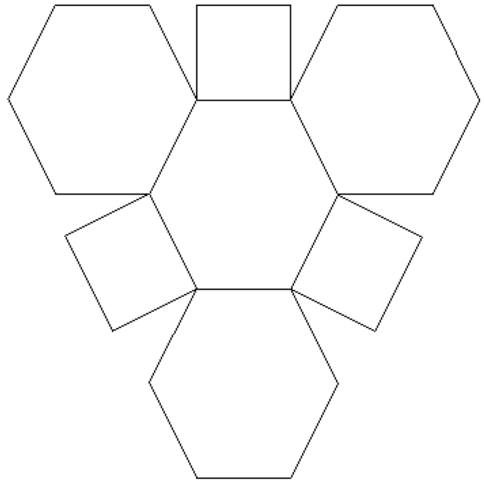


Figure 9.8

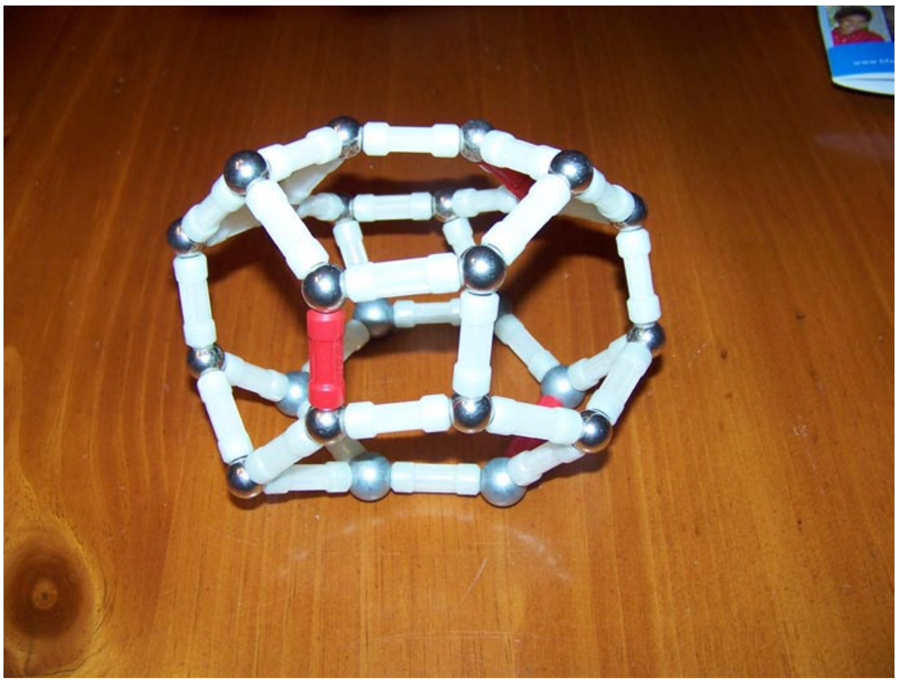


Figure 9.9

3. Truncated Hexahedron (or Truncated Cube)

The next solid in the Archimedean series is the truncated hexahedron, better known as a truncated cube. Cutting off the corners of a cube results in a new polygon face of eight sides called the octagon, and the resulting corner faces are all equilateral triangles. The truncated cube is made up of six octagons and eight equilateral triangles (**Figures 9.10 and 9.11**).

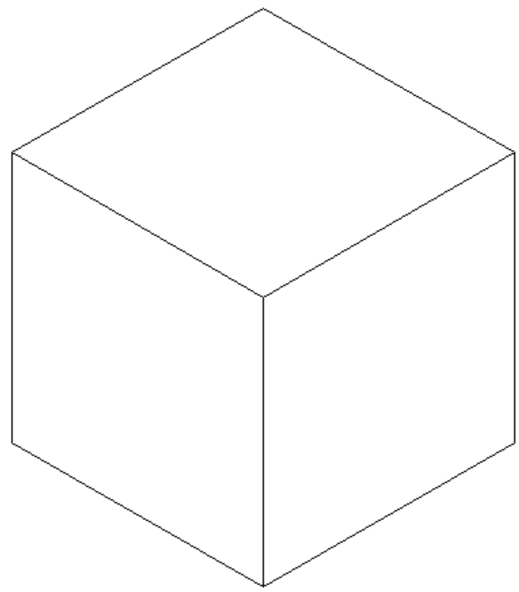


Figure 9.10

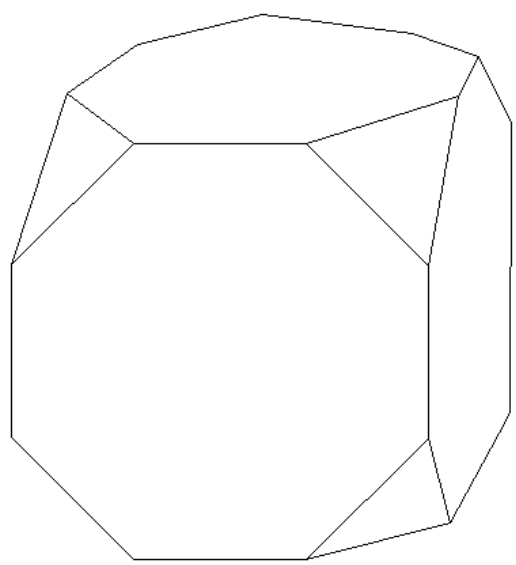


Figure 9.11

1. To make an octagon, begin by drawing a circle with a compass. Determine the diameter of the circle, line segment AB. Then construct a perpendicular CD through the center which will divide the circle into four quarters with center angles of 90 degrees each. (Review **Chapter 8**, Figure 8.15 to find the perpendicular to a line through a midpoint.)

2. Bisect each of these 90 degree angles. Open your compass to about 3” and place the point of the compass on point A. Sweep a small arc above point A. Without changing

the compass width, sweep another small arc from point C. Connect the intersection of these two arcs and the center of the circle O to bisect the angle and locate point G on the circle.

3. Repeat this process to locate points E, H, and F. The circle is now divided into 8 chords AG, GC, CE, EB, BH, HD, DF, and FA. Draw lines to indicate the 8 chords. See **Figure 9.12**.

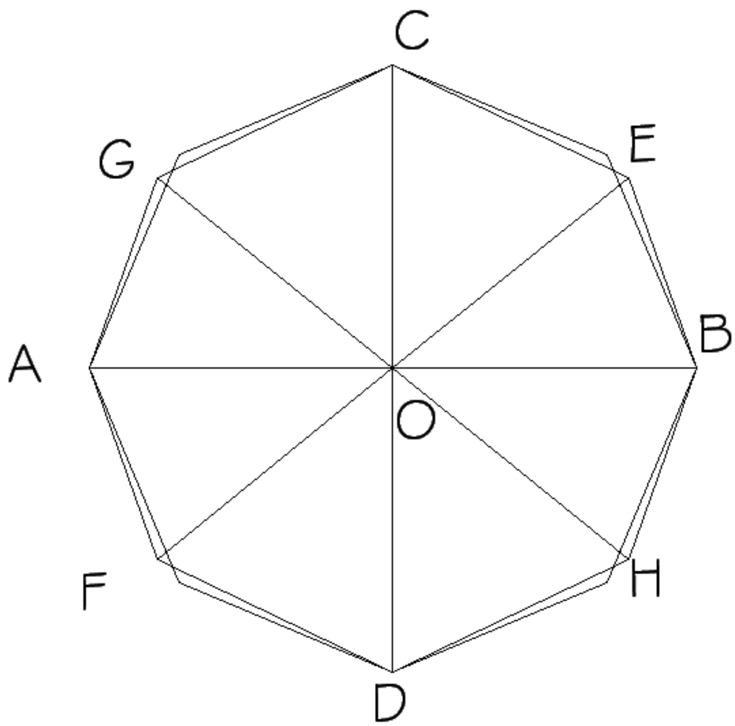


Figure 9.12

4. To construct an octagon to the specific size you have chosen, you can minimize or enlarge your octagon using the same principle we used to enlarge the pentagon in **Chapter 5**, Figures 5.8 and 5.9. Using the construction you have drawn above, extend any two connecting lines (or lines forming an angle) GC, CE, to the length of your choice, and then construct perpendiculars at their midpoints, which will intersect at a new center point, Z. See **Figure 9.13.**

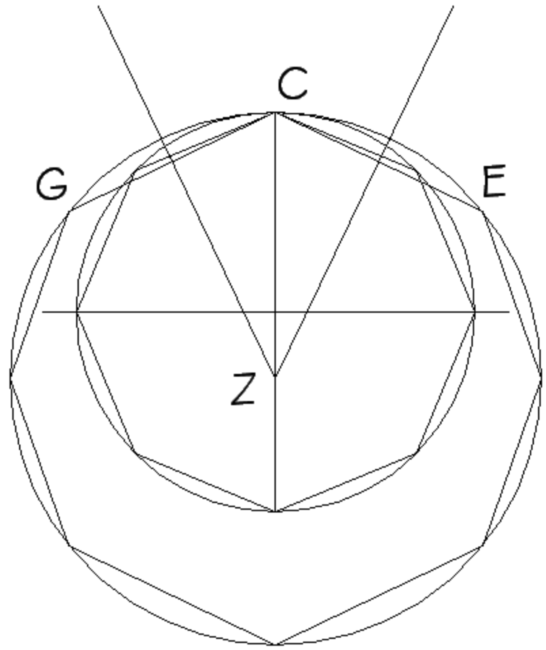


Figure 9.13

5. A circle drawn on this new point Z with a radius to angle C will enclose the two extended lines as chords. By setting your compass to the length of either one of these lines, you can sweep out the remaining six chords to complete a perfect equilateral octagon. Because of their greater size--even in the smallest scale--these octagons may have to be cut out of poster-board as individual pieces. We suggest that you make a total of 16 (17, if you wish to keep the template), since there are several other forms to be made using octagons.

6. The net for this shape, given in **Figure 9.14** may be so large that it will not fit your sheet of poster-board, requiring that you make the model in two or more parts. Lay out four octagons in a row, the first and third having equilateral triangles filling the lost corners. Above and below any of these octagons install two more, readily recalling the layout of the cube. One octagon should be left free to allow access to the interior for internal taping, after which you will have to seal the last octagon with tape on the exterior. The completed Truncated Hexahedron (or Truncated Cube) is shown in **Figure 9.15**.

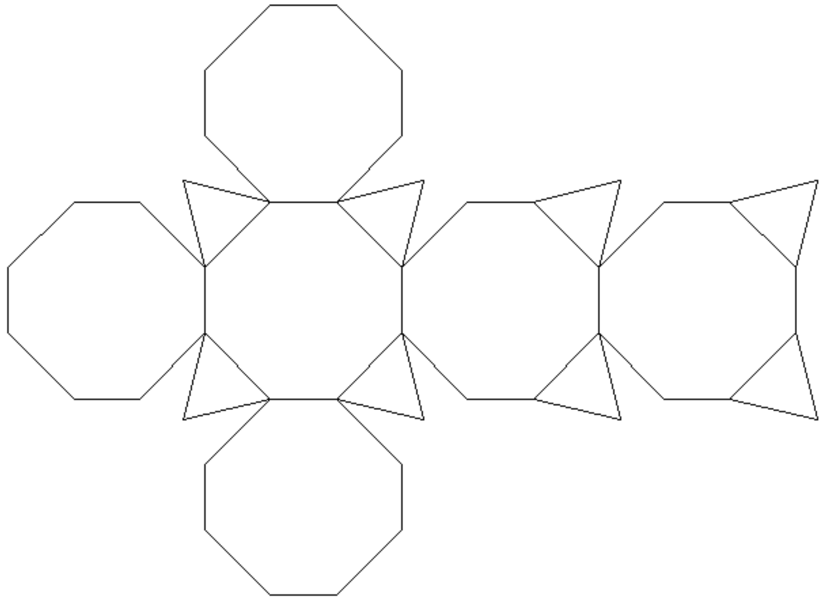


Figure 9.14

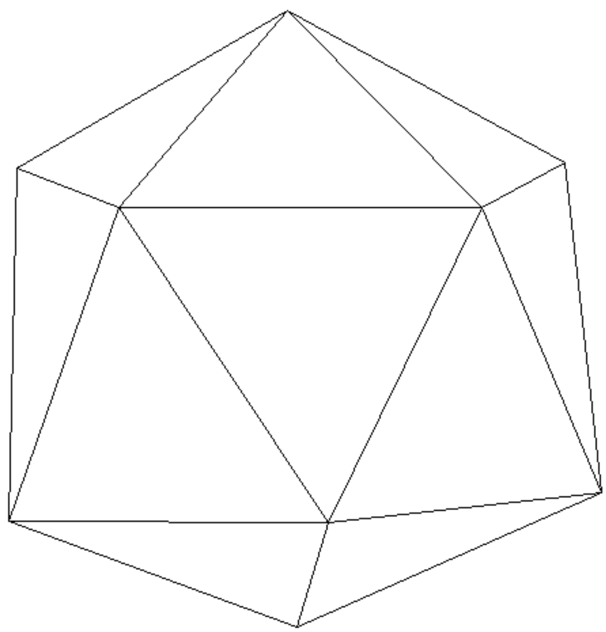


Figure 9.15

Additional Exercise:

What is the shape of the cut off portion of the truncated cube? While the cut off area of the truncated cube is an equilateral triangle, this is not true of the original square face. This would have been an isosceles triangle with a 90-degree angle. Three of these in pyramid form with an equilateral triangle base will provide one of the missing points of the truncated cube. While a tetrahedron of sorts, this shape should not be confused with the tetrahedron in the Platonic Solids series, which has four equilateral triangle faces.

4. Truncated Icosahedron

When finished, the truncated icosahedron bears a close resemblance to a soccer ball. In fact, the original design for the soccer ball--sometimes referred to as a 32-faced ball--consisted of a combination of 12 pentagons and 20 hexagons cut from leather. The same numbers and shapes apply to the truncated icosahedron.

1. The net for the truncated icosahedron is really quite simple, beginning with a pentagon surrounded by five hexagons. Each of these five hexagons has another hexagon attached to its outermost side and a pentagon attached to the adjacent side to the right. See **Figure 9.16**.

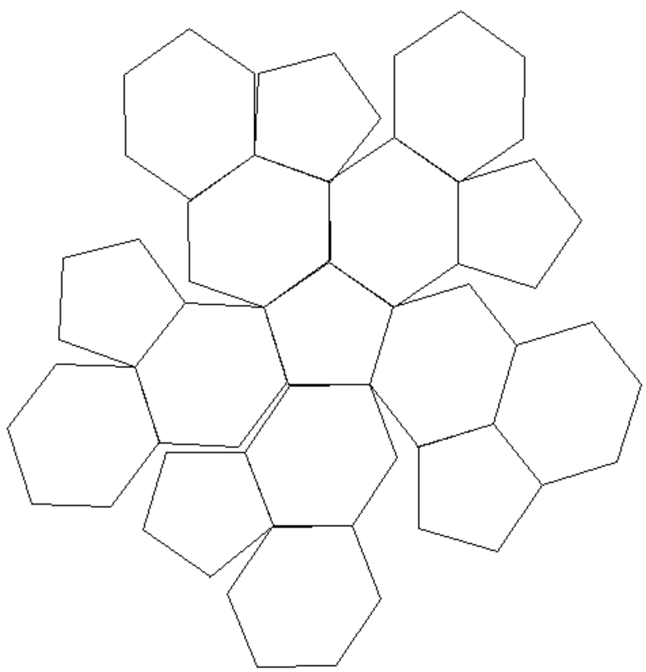


Figure 9.16

2. Full assembly of this net pattern will result in one-half of the construction. Repeat the process to complete the other half. Uniting the two halves will take some dexterity, but can be done by taping one common edge at a time. The finished product is shown in **Figure 9.17**.

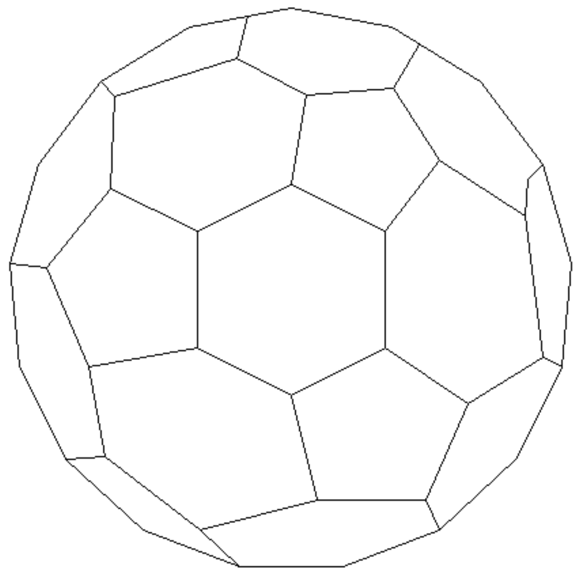


Figure 9.17

5. The Truncated Dodecahedron

Assembly of this shape requires the construction of a polygon which has not been introduced up to now, the decagon ("ten sided"). If you are determined to own the complete set of 13 Archimedean Solids, you will need at least 24 of these in all (25 with a template). Because these decagons are enormous, the large finished solid will be rather ungainly and fragile. It is recommended that in this case, you use a somewhat heavier poster stock. There is a lot of work required to complete this solid. It bears a close resemblance to the dodecahedron, except that all the corners are truncated. If you design it to scale with the side dimensions of all your other squares, triangles, etc., it will be big indeed!

1. Cutting off all the corners of a pentagon results in a decagon. Bisecting all of the sides of the pentagon will produce a regular decagon with ten equilateral sides--which is exactly what you want. Begin by following earlier instructions in **Chapter 6** to inscribe a pentagon in a circle--but make as large a circle as your compass will permit.

2. When pentagon ABCDE is complete, (see **Figure 9.18**), find the mid-point of each one of the five sides by drawing perpendiculars to them, cutting through each arc of the circle at FGHIJ. This can also be done by measuring the lengths of the chords and dividing by two. Connect points AF, FB, BG, GC, CH, etc. with chords to complete the decagon.

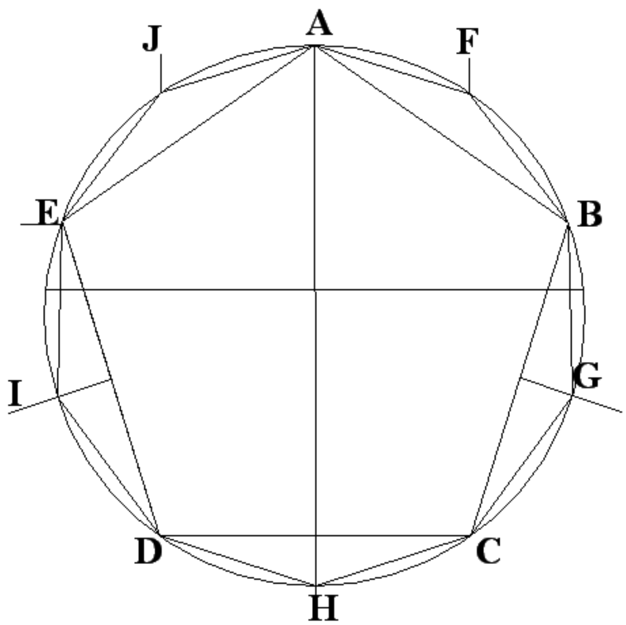


Figure 9.18

3. The decagon can by scaled up or down to the desired size using the principles in **Chapter 5**. You will need 12 decagons in all to complete this solid.

4. Having made your 12 decagons, lay one flat and surround it with five others on alternating sides.

5. Make 20 equilateral triangles with the same edge size as the decagons. Attach these triangles to the remaining sides of the original decagon. When these decagons are brought together, another five triangles can be installed in the grooves between them. This is half the assembly. The net provided in **Figure 9.19** is one half of the complete construction, and its resemblance to the dodecahedron is pronounced.

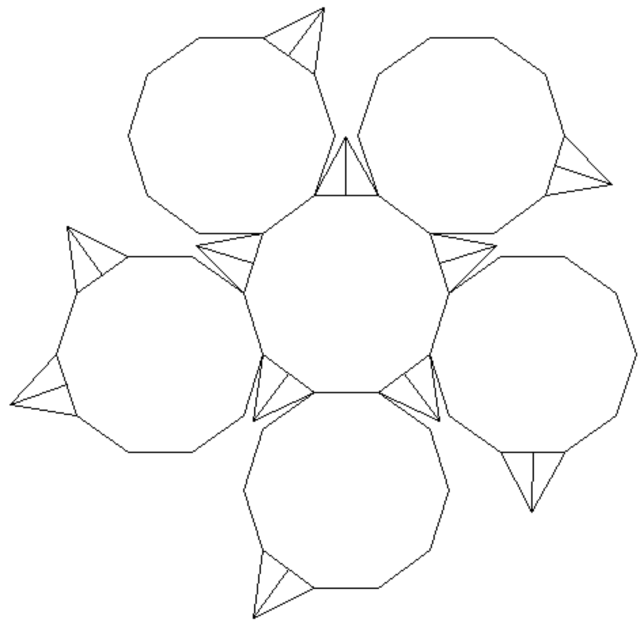


Figure 9.19

6. Another section just like this one will almost complete the solid--a band of five triangles will still be needed around the middle section. This completes the set of five truncated versions of the original 5 Platonic Solids. See **Figure 9.20**.

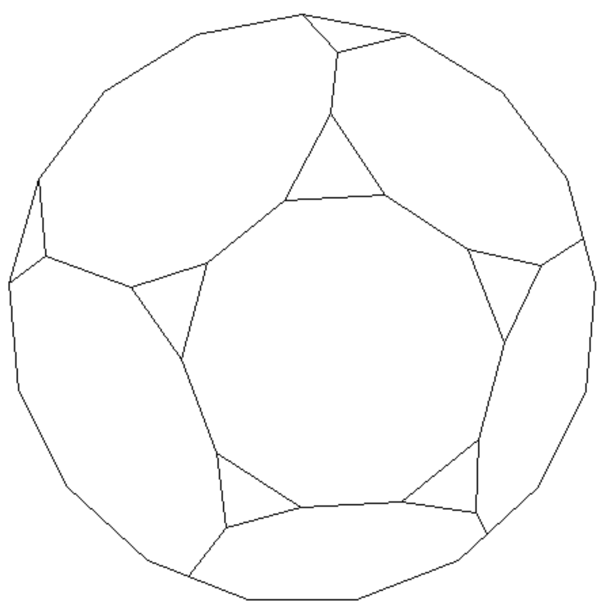
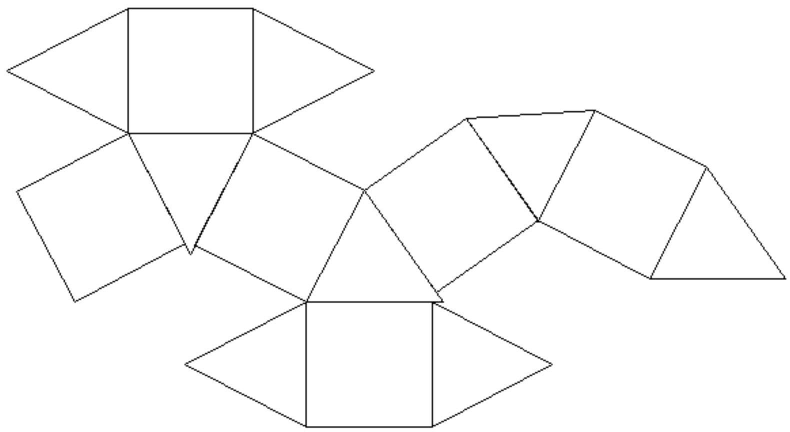


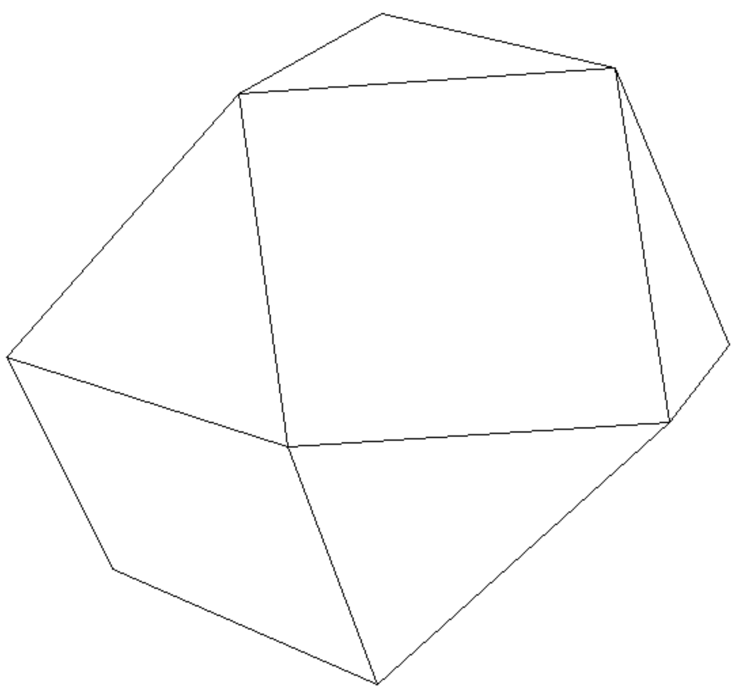
Figure 9.20

*** Nets and finished product diagrams are provided for the remaining 8 Archimedean Solids. You are encouraged to use the skills that you have acquired in making the first 5 solids to create the shapes needed to construct these solids.**

6. Cuboctahedron (Figures 9.21 - net and 9.22)

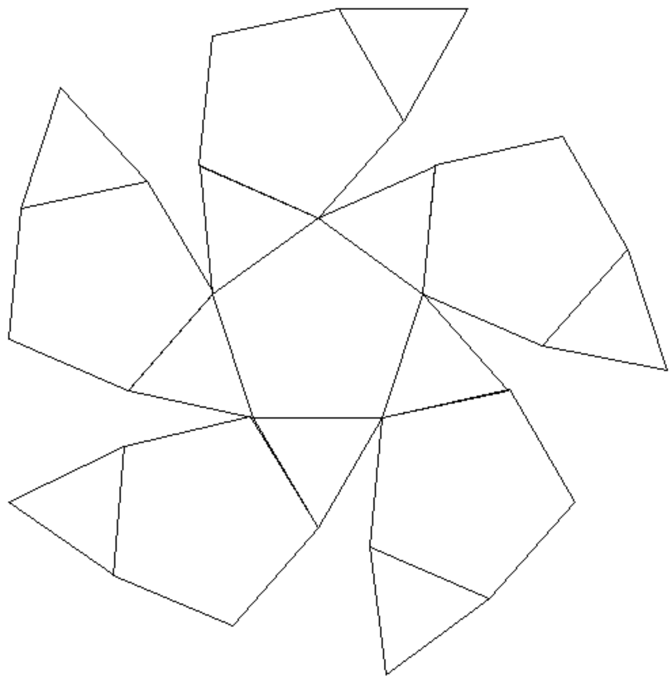


Figures 9.21

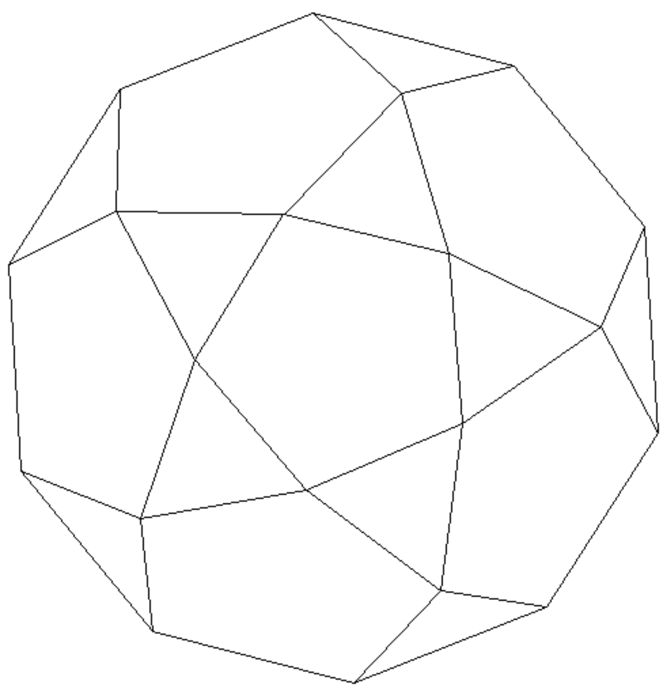


Figures 9.22

7. Icosadodecahedron (Figures 9.23 – net for one half of the solid and 9.24)

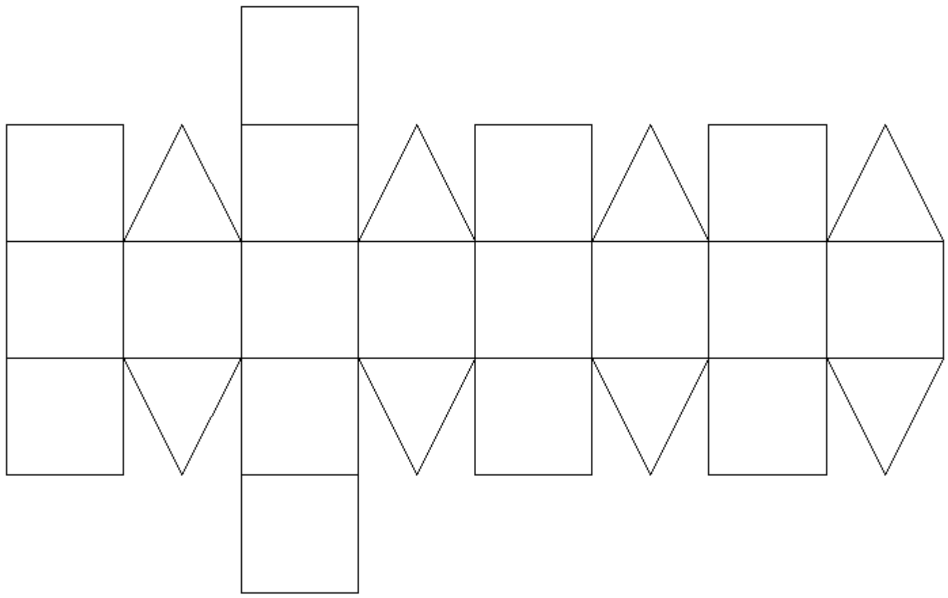


Figures 9.23

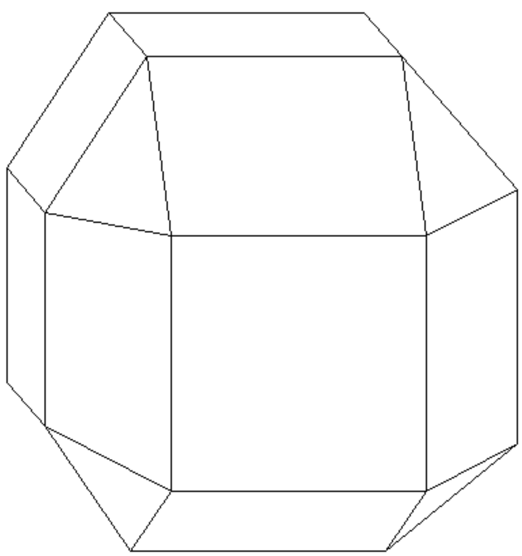


Figures 9.24

8. Rhombicuboctahedron (Figures 9.25 – net and 9.26)
(or Small Rhombicuboctahedron)

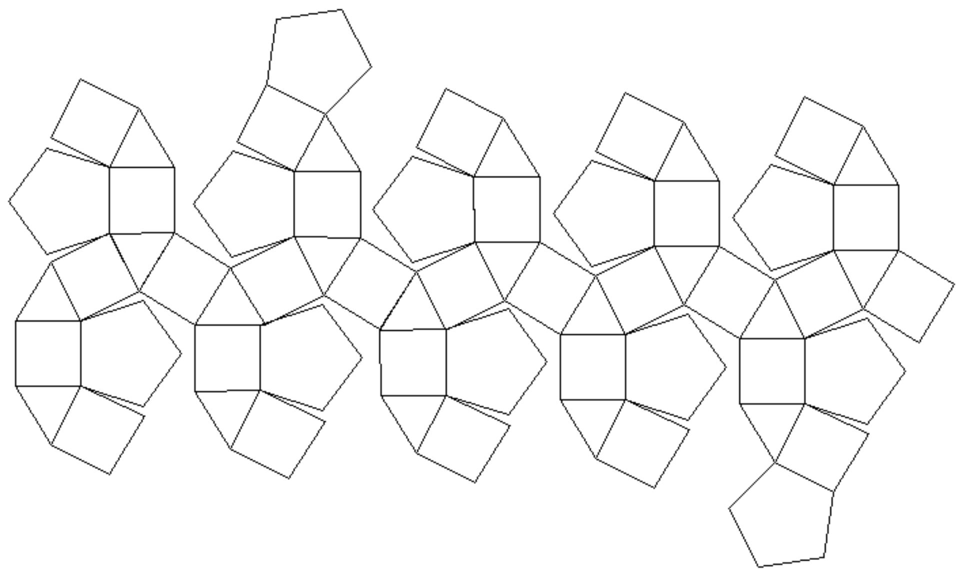


Figures 9.25

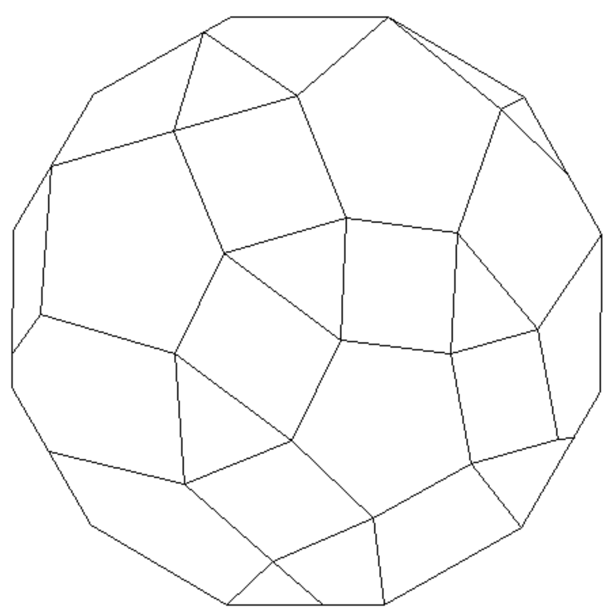


Figures 9.26

9. **Rhombicosidodecahedron (or Small Rhombicosidodecahedron)** **Figures 9.27 – net and 9.28**

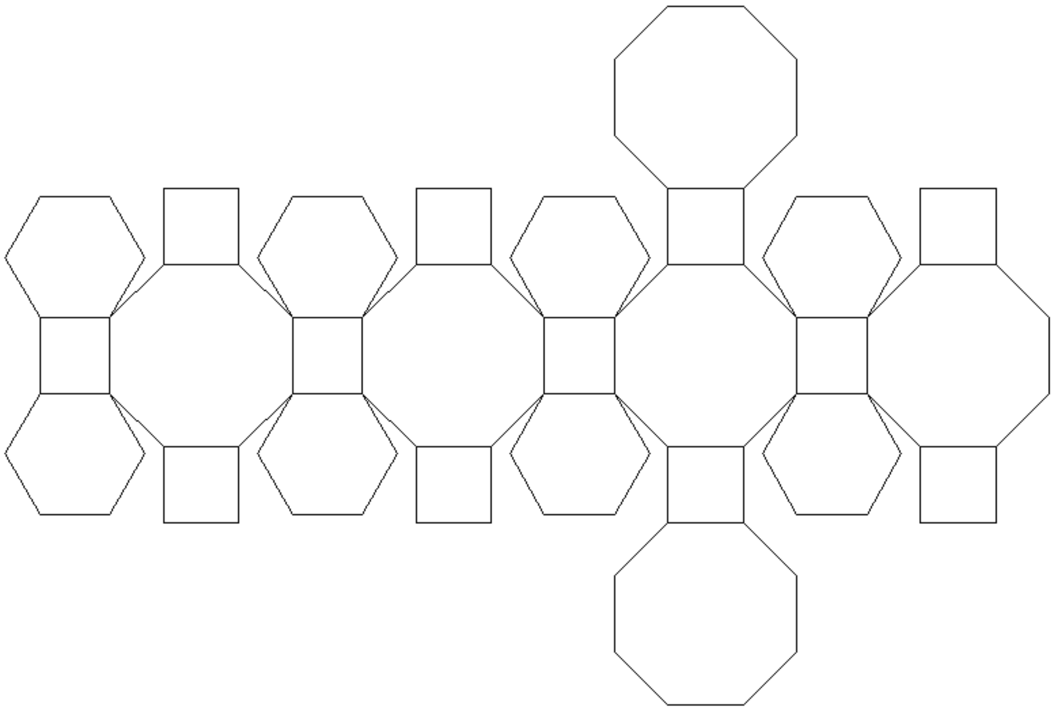


Figures 9.27

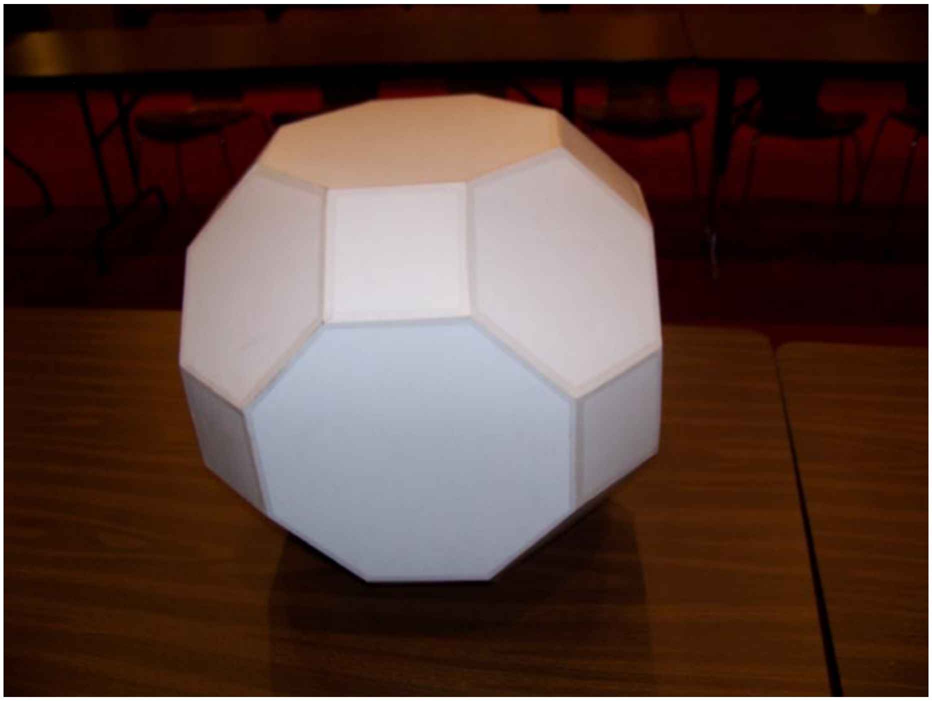


Figures 9.28

10. **Rhombitruncated Cuboctahedron (or Great Rhombicuboctahedron)** **Figures 9.29 – net and 9.30**

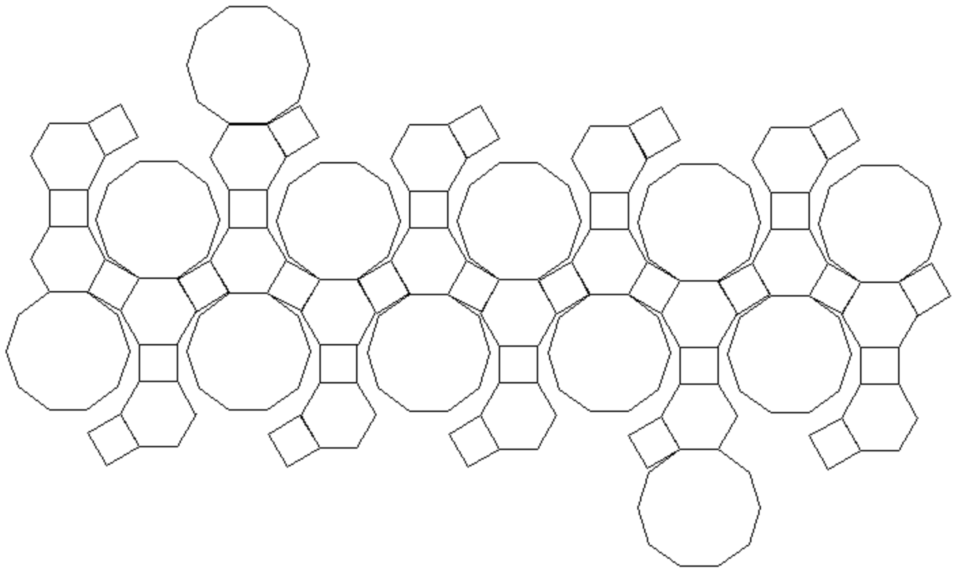


Figures 9.29

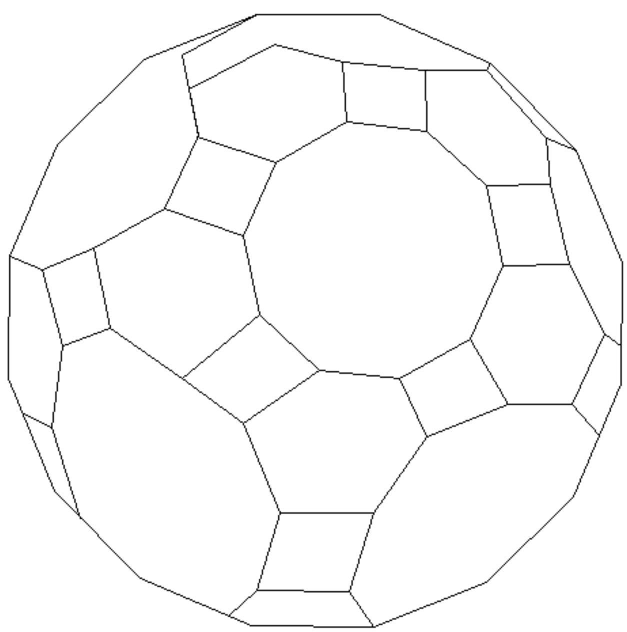


Figures 9.30

11. Rhombitruncated Icosidodecahedron (or Great Rhombicosidodecahedron) **Figures 9.31 – net and 9.32**

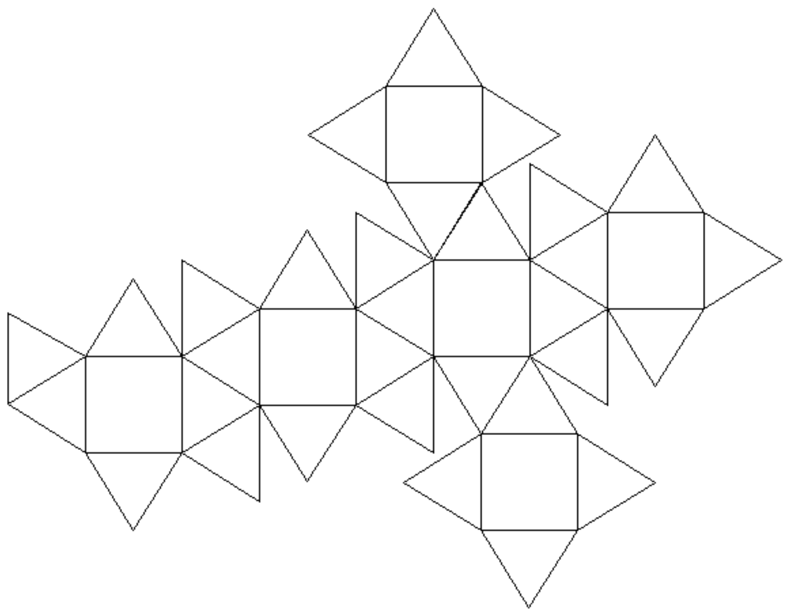


Figures 9.31

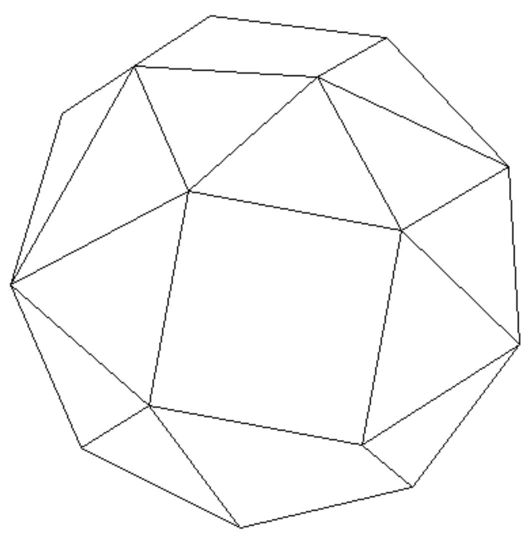


Figures 9.32

12. Snub Cube **Figures 9.33 – net and 9.34**

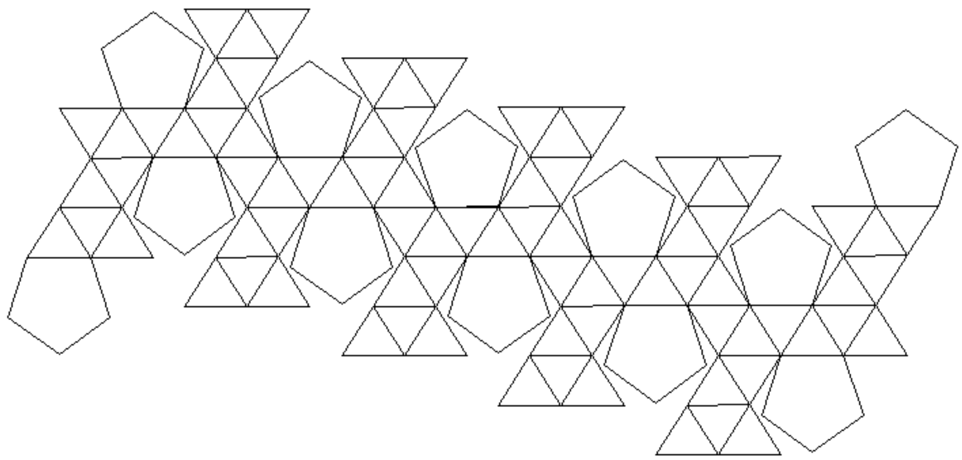


Figures 9.33

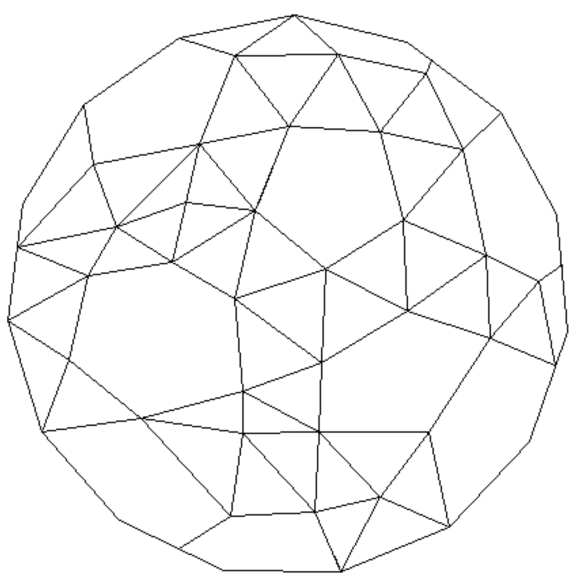


Figures 9.34

13. Snub Dodecahedron **Figures 9.35 – net and 9.36**



Figures 9.35



Figures 9.36

Well! You've successfully made your way through all of the Archimedean Solids, translating nets from a flat surface into a series of finished, three-dimensional solids. Many other forms remain to be explored and constructed, some of which we will create, while others go well beyond the scope of this publication.